INTRODUCTION

Background science contains information that is needed for S282 and S283. This science is not itself astronomy or planetary science, but underpins much of what you will be studying.

We hope you will dip into the topics as and when you need to — it is *not* the intention that you work through everything all in one go. Just have a quick look at the topics now to get an idea of the support that will be available to you as you work through your course.

You may already have met a good deal of the material presented here, particularly if you have studied the OU course S103 *Discovering Science* or one of its predecessors. *Background science* is intended to refresh your memory of key areas and to provide some basic information on topics that might be unfamiliar to you.

In addition to background science, we have also included basic mathematics. Topic 7 includes various items of useful information, such as the Greek alphabet and tables of physical constants and useful quantities. You will probably need to refer to these tables on many occasions during your study, particularly when working on assignments.

You can search for a particular word or phrase in the PDF files from the menu bar by selecting Edit | Find. In the text, key terms are printed in **bold** where they are introduced.

Each topic contains worked examples to illustrate the topic being discussed, and questions for you to test your own understanding. Worked solutions to all the questions are given at the end of each topic. In all the worked examples and solutions, we have stated the final answer with an appropriate number of significant figures, which generally involves rounding the number shown on a calculator display. Significant figures are explained in Section 6.2.

TOPIC 1
PHYSICAL QUANTITIES

1.1 Units

Your height, the distance to the Moon, the diameter of a tennis ball, are all examples of quantities that share the physical attribute of length. Length is an example of a physical quantity. Time, temperature and mass are three other types of physical quantity, and there are many more.

We can compare two physical quantities only if they are both of the same type — that is, if they have the same **dimensions**. For example, quantities such as your height, the distance to the Moon, the diameter of a tennis ball, the thickness of a human hair, all have dimensions of length and all can be expressed as multiples of one another (you can say that your height is so many times the diameter of a particular tennis ball). Quantities with the same dimensions can also be added together or subtracted from one another.

We cannot compare physical quantities of different dimensions. For example, we cannot say that your height is so many times your body temperature, or add the Moon's distance to its mass.

1.1.1 SI units

Each type of physical quantity can be measured in a variety of **units**. Thus, we can measure length in inches, furlongs, fathoms, miles, leagues, ångstroms, microns and of course, metres. In science the units used are known as **SI units**, which is an abbreviation for 'Système Internationale d'Unités' (International System of Units). This standard set of units was approved by an international conference in 1960. It is used world-wide, particularly in the scientific community because it avoids the need to convert laboriously from one system to another when comparing results. The SI unit of length is the metre and Table 1.1 lists all seven SI base units. (Notice that when a unit is named after a person the unit symbol has a capital letter but the full name of the unit does not.)

Table 1.1 The seven SI base units.

Physical quantity	SI unit and abbreviation
length	metre, m
time	second, s
mass	kilogram, kg
temperature	kelvin, K
electric current	ampere, A
luminous intensity	candela[1], cd
amount of substance	mole, mol

[1]Not commonly used but included here for completeness.

For quantities that are much larger or smaller than the standard SI unit, we use larger or smaller multiples either written as a power of ten or with one of the standard **SI prefixes** listed in Table 1.2. The official prefixes go up and down in steps of 10^3, but the centimetre (cm, $1\ \text{cm} = 10^{-2}\ \text{m}$) is also in common use.

When a prefix is put in front of a unit, it is important that there should be no gap between them, for example 15 milliseconds = 15 ms.

Table 1.2 Common SI prefixes.

Prefix	Symbol	Power of ten
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	μ*	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

* μ is the Greek letter mu.

In astronomy and planetary science you will also meet some non-SI units. Generally these are used to measure extremely large or small quantities, such as the distance between galaxies or the energy of a single electron. Some of the most common are listed in Topic 6 along with their SI equivalent.

1.1.2 Writing physical quantities

A physical quantity consists of a number *and* a unit. Without the unit, the quantity is incomplete. Thus speed = 23.4 tells us nothing, but speed = 23.4 m s^{-1} tells us its value. After all, 23.4 could have been mm s^{-1}, or mph, or anything else. When a symbol represents a quantity, it represents the complete quantity, units and all. For example, using v to represent speed, we could write

$$v = 23.4\ \mathrm{m\,s^{-1}}$$

It is incorrect to write just v = 23.4, or v (m s^{-1}) = 23.4. Nor should you refer to 'a speed of v m s^{-1}', because v represents the speed complete with its units.

> Note that kilograms are written kg *not* kgm. For similar reasons note that plural units are *not* followed by an 's' as that would be confused with seconds. So a length of fifteen metres is written 15 m (*not* 15 ms as that would mean fifteen milliseconds).

Units can be manipulated just like numbers or any other symbol. When labelling the *axes* of *graphs*, and when listing physical quantities in tables, it is conventional to divide each quantity by its unit to get a pure number, that is a **dimensionless quantity**. For example, you can divide both sides of the expression above by m s^{-1} and write

$$\frac{v}{\mathrm{m\,s^{-1}}} = 23.4$$

If you were plotting values of v on a graph, or listing them in a table, you should label the graph axis, or the table column, as v/m s^{-1}. Figure 1.1 gives an example of a graph with correctly labelled axes.

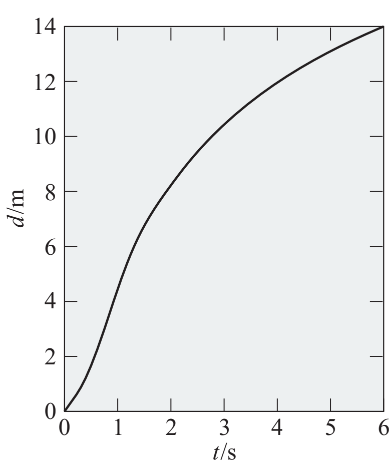


Figure 1.1 The distance d a body travels in time t.

Suppose you were dealing with speeds that were all several million metres per second, written in scientific notation:

$$v = 1.23 \times 10^{6}\ \mathrm{m\,s^{-1}}$$

$$v = 3.45 \times 10^{6}\ \mathrm{m\,s^{-1}},\ \text{etc.}$$

To make the numbers more manageable, you can divide each speed by 10^6 m s^{-1} and write

$$\frac{v}{10^{6}\ \mathrm{m\,s^{-1}}} = 1.23\ \ \text{etc.}$$

QUESTION 1.1

Suppose the speed v = 1.23 × 10^{6} m s^{-1} was measured at a time of t = 6.7 × 10^{-3} s, and v = 3.45 × 10^{6} m s^{-1} was measured at a time t = 8.9 × 10^{-3} s. Draw up a table with suitably headed columns for recording these values.

To read values from graphs or tables, you simply 'undo' the way they are written, as in the following example.

Table 1.3 lists some data about radioactive isotopes heating the Earth. From this table, the half-life of ^{235}U is found by writing:

$$\frac{\text{half-life}}{10^9\ \text{yr}} = 0.71$$

where 'yr' is an abbreviation for year, so half-life = 0.71×10^9 yr = 7.1×10^8 yr.

For periods of time of millions of years, we use the units Ma (1Ma = 1×10^6 yr) or Ga (1 Ga = 1×10^9 yr).

QUESTION 1.2

From Table 1.3, what is the rate of heating by ^{238}U?

1.2 Manipulating units

1.2.1 Combining units

Table 1.3 Heating by uranium isotopes per kg of the Earth's mass.

Isotope	Half-life/10^9 yr	Rate of heating/10^{-12} W kg^{-1}
^{235}U	0.71	0.04
^{238}U	4.50	0.96

You can combine two quantities by multiplying or dividing one by the other and this produces a quantity with new units. Units can be multiplied or divided just like any other symbols.

EXAMPLE 1.2

A wall is 3.20 m high and 4.50 m wide. What is its area?

To find its area, multiply the height and width together:

$$\text{area} = 3.20\,\text{m} \times 4.50\,\text{m} = 14.4\,\text{m}^2$$

The SI units of area are metres × metres, that is square metres (m^2).

EXAMPLE 1.3

Walking to the corner shop, 480 metres away, takes 4 minutes (i.e. 240 seconds). What is the average speed of the journey?

To find the average speed for the journey, divide the distance travelled by the time taken:

$$\text{speed} = \frac{480\,\text{m}}{240\,\text{s}} = 2.0\,\text{m}\,\text{s}^{-1}$$

The answer has SI units of $\frac{\text{metres}}{\text{seconds}}$, abbreviated to m/s or m s^{-1}.

QUESTION 1.3

For each of the following, write down the type of physical quantity and the SI unit:

(i) the volume of the Earth,

(ii) the age of the Earth,

(iii) the ratio of our distance from the Moon to our distance from the Sun.

Some common combinations of SI base units occur so often that they are given their own name and symbol.

The SI unit of *force* is defined, via the equation $F = ma$, as the force required to give a mass of 1 kg an acceleration of $1\ \mathrm{m\,s^{-2}}$. This force unit is called the newton, N, after the British scientist Isaac Newton (1642–1727). So putting $m = 1$ kg and $a = 1\ \mathrm{m\,s^{-2}}$ gives

$$1\ \mathrm{N} = 1\ \mathrm{kg} \times 1\ \mathrm{m\,s^{-2}} = 1\ \mathrm{kg\,m\,s^{-2}}$$

QUESTION 1.4

The SI unit of work and also energy is defined, via the equation $W = Fd$, as the work, W, done when a force of $F = 1$ N moves something through a distance of $d = 1$ m. This unit is called the joule, J, after the British scientist James Joule (1818–1889). Express the joule in terms of SI base units.

1.2.2 Getting the right units

When writing equations, the two things either side of the equals sign are always exactly the same in all respects — not only the same number but also the same units. You can use this to deduce the units of new quantities.

Newton's law of gravitation states that the force of attraction, F, between two objects of masses M and m, separated by a distance r, is given by the equation:

$$F = \frac{GMm}{r^2}$$

where G is the gravitational constant. To find the SI units of G, first rearrange the equation to make G the *subject*, i.e. G is isolated on one side of the equation, usually the left, so

$$G = \frac{Fr^2}{Mm}$$

The units of G must be those of the right-hand side of the equation. Using the newton, the SI unit of force, we can write

$$\begin{aligned}\text{units of } G &= \frac{(\text{units of } F) \times (\text{units of } r^2)}{(\text{units of } M) \times (\text{units of } m)} \\ &= \frac{\mathrm{N} \times \mathrm{m}^2}{\mathrm{kg} \times \mathrm{kg}} \\ &= \mathrm{N\,m^2\,kg^{-2}}\end{aligned}$$

QUESTION 1.5

At temperature T, the average atomic kinetic energy, E_k, is given by the expression:

$$E_k = \frac{3kT}{2}$$

where k is the Boltzmann constant. Deduce the SI units of k.

Fundamental constants, such as the Boltzmann constant, are parameters that do not change throughout the Universe.

1.2.3 Checking equations

Sometimes you manipulate some algebra to derive a new equation. A good way to check your result is to compare the units on the left- and right-hand sides. They should be the same — and if they are not, your equation cannot be correct.

EXAMPLE 1.4

Suppose you derived an expression for the wavelength, λ, of a wave with frequency f travelling at speed v, and wrote down

$$\lambda = \frac{v}{f}$$

Frequency has SI units of hertz, Hz (1 Hz = 1 s^{-1}). Can the equation be correct?

$$\text{units of right-hand side} = \text{m s}^{-1}/\text{s}^{-1} = \text{m}$$

As wavelength (being a length) has SI units of metres, this gives the units that we expect for the left-hand side, which is reassuring. (The equation is in fact correct.)

EXAMPLE 1.5

Suppose you derived an equation for the minimum speed, v, that would enable an object to escape from a planet of mass M and radius R and wrote down:

$$v = \frac{2GM}{R}$$

where G is the gravitational constant.

G has SI units N m^2 kg^{-2}, and 1 N = 1 kg m s^{-2}. Can the equation be correct?

Remembering that the 2 in the numerator is dimensionless, we have for the right-hand side (rhs)

$$\begin{aligned}\text{units of rhs} &= \frac{(\text{units of } G) \times (\text{units of } M)}{(\text{units of } R\,)} \\ &= \frac{\text{N m}^2\ \text{kg}^{-2} \times \text{kg}}{\text{m}} \\ &= \frac{\text{kg m s}^{-2} \times \text{m}^2\ \text{kg}^{-2} \times \text{kg}}{\text{m}} \\ &= \text{m}^2\ \text{s}^{-2}\end{aligned}$$

Now look at the left-hand side. The SI units of speed are $m\,s^{-1}$ not $m^2\,s^{-2}$, so this equation *cannot* be correct. The equation should in fact be:

$$v = \sqrt{\frac{2GM}{R}}$$

so the units on the rhs are $\sqrt{m^2\,s^{-2}} = m\,s^{-1}$ as required.

QUESTION 1.6

Suppose you derived an expression for the energy E of a single photon of light with wavelength λ and wrote down

$$E = hc\lambda$$

where c is the speed of light and h is the Planck constant (6.63×10^{-34} J s). Could the equation be correct?

QUESTION 1.7

It is possible to deduce the mass of a large object (e.g. a star or planet) by timing the motion of a much smaller object (e.g. planet or moon) in orbit around it. Suppose you derived an expression for the mass M of the large object in terms of the orbital radius r and the time, P, for one orbit, and found

$$M = \frac{2\pi r^3}{GP^2}$$

where G is the gravitational constant. G has SI units $N\,m^2\,kg^{-2}$, and $1\,N = 1\,kg\,m\,s^{-2}$. Can the equation be correct? (*Note*: 2π is dimensionless.)

1.2.4 Converting between units

Before starting a calculation it is normally advisable to put all quantities into standard SI units with no prefixes (but remember that the SI base unit of mass is the kilogram not the gram). Jotting down your conversion as in the following examples can help you avoid mistakes.

These examples use standard scientific 'powers of ten' notation. See Topic 6.

EXAMPLE 1.6

Jupiter's radius is 7.14×10^4 km. What is that in metres?

$1\,km = 1 \times 10^3\,m$ (i.e. $10^3\,m$), so radius $= 7.14 \times 10^4 \times 10^3\,m = 7.14 \times 10^7\,m$.

EXAMPLE 1.7

The red light emitted by hydrogen atoms has a wavelength 656 nm. What is that in metres?

$656 = 6.56 \times 10^2$, and $1\,nm = 1 \times 10^{-9}\,m$, so wavelength $= 6.56 \times 10^2 \times 10^{-9}\,m$ $= 6.56 \times 10^{-7}\,m$.

EXAMPLE 1.8

A piece of wire has a cross-sectional area of 1 mm^2. What is that in square metres (i.e. m^2)?

$1\ mm^2 = 1\ mm \times 1\ mm$, and

$1\ mm = 1 \times 10^{-3}\ m$ (i.e. 10^{-3} m), so

$1\ mm^2 = 10^{-3}\ m \times 10^{-3}\ m = 10^{-6}\ m^2$

EXAMPLE 1.9

Suppose a car travels at an average speed of 72 km per hour (72 km h^{-1}). What is that speed in metres per second?

$$72\ \text{km h}^{-1} = \frac{72\ \text{km}}{1\ \text{h}}$$

$1\ km = 1 \times 10^3\ m$, and $1\ h = 3600\ s$, so

$$72\ \text{km h}^{-1} = \frac{72 \times 10^3\ \text{m}}{3600\ \text{s}} = \frac{720\ \text{m}}{36\ \text{s}} = 20\ \text{m s}^{-1}$$

EXAMPLE 1.10

A rock sample has volume 64 cm^3 and mass 0.16 kg. What is its volume in m^3? What is its density (mass per unit volume) in kg m^{-3}?

$1\ cm = 1 \times 10^{-2}\ m$ (i.e. 10^{-2} m) so

$1\ cm^3 = 10^{-2}\ m \times 10^{-2}\ m \times 10^{-2}\ m$

$= 10^{-6}\ m^3$ and so

volume $= 64 \times 10^{-6}\ m^3 = 6.4 \times 10^{-5}\ m^3$

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{0.16\ \text{kg}}{6.4 \times 10^{-5}\ \text{m}^3} = 2.5 \times 10^3\ \text{kg m}^{-3}$$

QUESTION 1.8

In a vacuum, light travels at 6.706×10^8 miles per hour. Given that 1 mile is 1609 metres and 1 hour is 3600 seconds, calculate the speed of light in SI units.

QUESTION 1.9

The volume of the Earth is $1.083 \times 10^{12}\ km^3$. What is its volume in m^3? If the average density of the Earth is 5.51×10^3 kg m^{-3}, what is its mass?

1.3 Answers and comments for Topic 1

Table 1.4 The answer to Question 1.1.

Time $t/10^{-3}$ s	Speed $v/10^{6}$ m s^{-1}
6.7	1.23
8.9	3.45

QUESTION 1.1

The columns could be headed $t/10^{-3}$ s and $v/10^{6}$ m s^{-1}. See Table 1.4. You could also use SI prefixes and head the t column t/ms.

QUESTION 1.2

From Table 1.3, rate of heating/10^{-12} W kg^{-1} = 0.96

so rate of heating = 0.96×10^{-12} W kg^{-1} = 9.6×10^{-13} W kg^{-1}.

QUESTION 1.3

(i) volume, m^3; (ii) time, s; (iii) dimensionless (one length divided by another, so the SI units are m/m, i.e. the units cancel so there are no units).

QUESTION 1.4

If F = 1 N = 1 kg m s^{-2} and d = 1 m, then 1 J = 1 kg m s^{-2} × 1 m = 1 kg m^2 s^{-2}.

QUESTION 1.5

First rearrange the equation to make k the *subject*:

$$k = \frac{2E_\mathrm{k}}{3T}$$

Using the joule, the SI unit of energy, and ignoring the numbers 2 and 3 since they have no units, we can write

$$\text{units of } k = \frac{\text{units of } E_\mathrm{k}}{\text{units of } T} = \frac{\mathrm{J}}{\mathrm{K}} = \mathrm{J\,K^{-1}}$$

QUESTION 1.6

The speed c has SI units m s^{-1}, so

$$\text{units of rhs} = \mathrm{J\,s} \times \mathrm{m\,s^{-1}} \times \mathrm{m} = \mathrm{J\,m^2}$$

The SI units of the lhs are J, so this equation *cannot* be correct. (The correct expression is in fact $E = hc/\lambda$. The rhs then has SI units of J as required.)

QUESTION 1.7

Before working out the units of the rhs it is helpful to deal with the units of GP^2 separately.

$$\begin{aligned}\text{units of } GP^2 &= \mathrm{N\,m^2\,kg^{-2}} \times \mathrm{s^2}\\ &= \mathrm{kg\,m\,s^{-2}} \times \mathrm{m^2\,kg^{-2}} \times \mathrm{s^2}\\ &= \mathrm{kg^{-1}\,m^3}\end{aligned}$$

So now we have units of rhs = m^3/(kg^{-1} m^3) = kg. As mass, M, on the lhs of the equation has SI units of kg, this is exactly what we need for the equation to be correct (which indeed it is).

QUESTION 1.8

Speed = $6.706 \times 10^8 \times 1609$ m/3600 s = 2.997×10^8 m s^{-1}

QUESTION 1.9

1 km = 1×10^3 m, so 1 km^3 = 1 km × 1 km × 1 km = 10^3 m × 10^3 m × 10^3 m = 10^9 m^3.

$$\begin{aligned}\text{Volume of Earth} &= 1.083 \times 10^{12}\ \text{km}^3 \\ &= 1.083 \times 10^{12} \times 10^9\ \text{m}^3 \\ &= 1.083 \times 10^{21}\ \text{m}^3.\end{aligned}$$

Mass = density × volume (see Example 1.10)

$$= 5.51 \times 10^3\ \text{kg m}^{-3} \times 1.083 \times 10^{21}\ \text{m}^3 = 5.97 \times 10^{24}\ \text{kg}.$$

TOPIC 2
MOTION, FORCE AND ENERGY

2.1 Newton's laws of motion

2.1.1 Speed, velocity and acceleration

When something is moving, there are several questions we can ask about its motion. How fast is it travelling? Which way is it going? Is it getting faster or slower? Is it changing direction?

The answer to 'how fast …?' can be found using

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}} \qquad (2.1\text{a})$$

In symbols, with u the speed, d the distance travelled and t the time taken:

$$u = \frac{d}{t} \qquad (2.1\text{b})$$

If the **speed** is constant, then equal distances are covered in equal times and a graph of distance d against time t is a straight line. Figure 2.1 shows a graph of d against t for a car travelling at a constant speed of $20\,\text{m s}^{-1}$, that is in each second, its distance increases by 20 m.

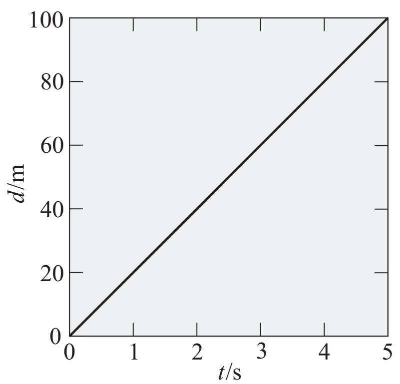


Figure 2.1 A graph of distance against time for a car travelling at a constant speed of $20\,\text{m s}^{-1}$.

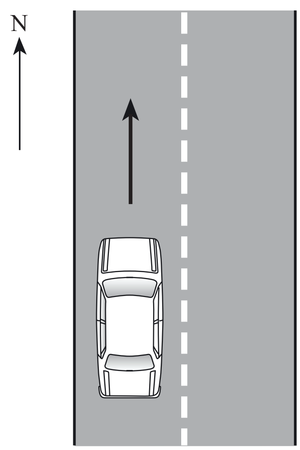


Figure 2.2 A car heading due north.

Speed alone does not fully describe the car's motion. Figure 2.2 shows the car heading due north along a straight road. A car travelling due south at $20\,\text{m s}^{-1}$ has the same speed (and the same graph of d against t) but its motion is different because it is moving in a different direction. To describe the motion fully we need to state both the car's speed and its direction; these two things together specify the **velocity**. So we can say that the car has a velocity of $20\,\text{m s}^{-1}$ due north.

An object's **acceleration** is the rate of change of its velocity. The car in Figure 2.2 can change its velocity by changing its speed. If it speeds up to, say, $22\,\text{m s}^{-1}$ it is accelerating in the everyday sense of 'getting faster'. If it reduces its speed to, say, $18\,\text{m s}^{-1}$, then in everyday terms we would say it is decelerating (getting slower) but scientifically we use the word 'acceleration' to cover all changes of velocity regardless of whether something is speeding up or slowing down.

Now suppose the car travels round a bend at a constant speed of 20 m s^{-1}. Its direction is changing — it is no longer heading due north — and so its velocity is changing. The car is accelerating (in the scientific sense) even though it is neither speeding up nor slowing down.

QUESTION 2.1

Figure 2.3 shows the paths of four accelerating objects a–d. The dots represent the positions of the objects at equal time intervals and the arrow indicates the direction of motion. For each diagram, say whether the object is speeding up, slowing down and/or changing direction.

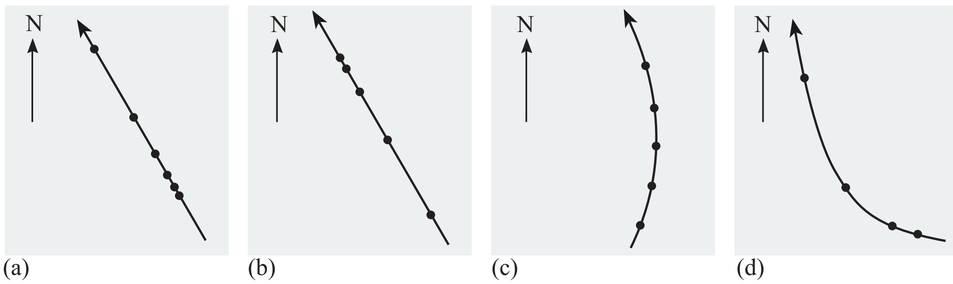


Figure 2.3 Paths of accelerating objects.

Acceleration can be calculated:

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}} \tag{2.2a}$$

In symbols, with u the initial velocity and v the velocity after a time interval t, then the acceleration a is given by

$$a = \frac{v - u}{t} \tag{2.2b}$$

EXAMPLE 2.1

Suppose the car described above takes 3 seconds to accelerate from 20 m s^{-1} to 26 m s^{-1}. What is its acceleration?

Change in velocity = initial velocity – final velocity

$$= 26\text{ m s}^{-1} - 20\text{ m s}^{-1} = 6\text{ m s}^{-1}$$

so

$$\text{acceleration} = \frac{6\text{ m s}^{-1}}{3\text{ s}} = 2\text{ m s}^{-2}$$

Notice that the SI units of acceleration are m s^{-2}, which can be read as metres per second, per second.

If the speed decreases, then the acceleration is negative. (Calculations of acceleration when direction is changing are not required for this course. Such calculations involve maths beyond the scope of S282, S283 and this booklet.)

QUESTION 2.2

A person pushing a wheelbarrow takes 3 seconds to reach a steady walking pace of 1.5 m s^{-1}, starting from rest. What is their acceleration?

2.1.2 Motion at constant velocity

If an object is at rest it will remain at rest unless anything disturbs it and makes it move, and if a moving object is undisturbed — nothing pushing or pulling on it, no friction, nothing to get in the way of its motion — it continues with constant velocity. Velocity can only change if a **force** acts on the object. The general definition of force is closely linked to acceleration: a force is that which causes acceleration. If no force acts, then velocity remains constant. Conversely, if velocity is constant, then there must be no force acting.

We have to be a careful what we mean by ‘no force’. We really mean ‘no unbalanced force’. It is possible to have two or more forces acting on a body and cancelling one another out. Force, like velocity, has both a size (a magnitude) and a direction. For two forces to cancel, they must not only have equal magnitudes but act in opposite directions. For example, if you are pushing a car on a level road the force you exert is countered by frictional forces between the car’s moving parts, as in Figure 2.4. If the frictional force exactly balances your muscular force (when the car is either at rest or in motion), the two forces cancel one another and there is no net force acting — there is no unbalanced force.

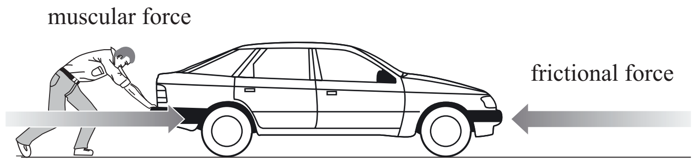


Figure 2.4 Two forces acting on a car that is being pushed.

This important result is summarized in **Newton’s first law of motion**:

> An object does not accelerate unless it is acted on by an unbalanced force.
>
> Equivalently: if an object is acted on by an unbalanced force it accelerates.

QUESTION 2.3

After some initial downward acceleration, a skydiver falls towards the Earth at constant speed. What can you deduce about any forces acting on the skydiver while she is falling vertically at constant speed?

2.1.3 Force, mass and acceleration

If, when pushing a car, your muscular force exceeds the frictional force then there is an unbalanced force and the car accelerates in the direction of that force. For a given object, the greater the unbalanced force acting on it, the greater its acceleration. But if the same force acts on objects of different mass, it produces different accelerations: the greater the mass, the smaller the acceleration produced by a given force. This enables us to define what we mean by **mass**: loosely speaking, mass is a measure of 'reluctance to accelerate'. If the mass is doubled, the acceleration produced by a given force halves, so that the product of 'mass times acceleration' stays the same.

This is an expression of **Newton's second law of motion**, and it can be stated in symbols:

$$F = ma \tag{2.3}$$

where F is the magnitude of the unbalanced force and a the acceleration it gives to a mass m.

From Newton's second law we can define a unit of force: in SI units, the unit of force (the newton, N) is the unit of mass (kg) multiplied by the unit of acceleration (m s^{-2}).

EXAMPLE 2.2

A car accelerates at 1.5 m s^{-2}. If the mass of the car and its occupants is 800 kg, what is the unbalanced force acting on the car?

Using $F = ma$, the unbalanced force is given by

$$F = 800\text{ kg} \times 1.5\text{ m s}^{-2} = 1200\text{ N} = 1.2 \times 10^3\text{ N}$$

QUESTION 2.4

Suppose that a person is sitting on a sledge on a horizontal icy surface, where the friction between the sledge and the ice is negligible. The combined mass of the person and sledge is 80 kg. What is the magnitude of the steady force that you need to apply, to accelerate the person and sledge in a straight line so that their speed increases from zero to a moderate walking pace (1.5 m s^{-1}) in 10 seconds?

QUESTION 2.5

A wind blowing on an oil-tanker of mass 2.0×10^8 kg exerts an unbalanced force of 4.0×10^6 N. What is the resulting acceleration?

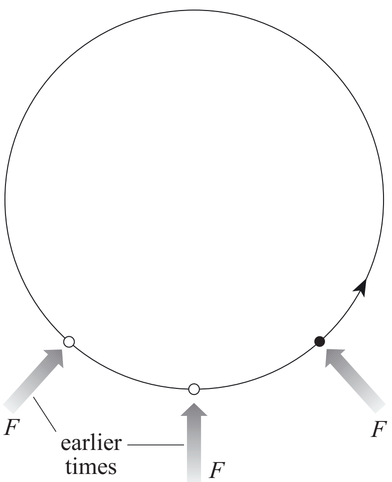


Figure 2.5 A force of constant magnitude, acting at right angles to the direction of motion, produces motion in a circle.

2.1.4 Motion in a circle

The examples (above) of pushing cars and sledges are all concerned with forces that act along, or against, the direction of motion and cause a change of speed. But what sort of force can produce an acceleration that is only a change of direction? Figure 2.5 shows how an unbalanced force can produce motion in a circle. The force is always at right angles to the direction of motion at a given instant, otherwise it would cause the speed, as well as direction, to change. Notice that the force is directed inwards, towards the centre of the circle.

The force causing circular motion can be the result of a single force, or an imbalance resulting from the inward force being greater than the outward force. Note that it is the force that causes the circular motion, not the other way around. If the force suddenly stops acting, then the object continues to move in a straight line as shown in Figure 2.6. So if you observe something moving in a circle, you know that there *must*

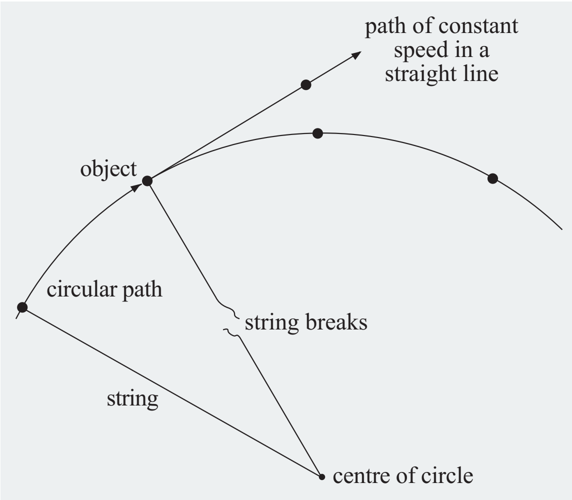


Figure 2.6 The effect on circular motion if the inward-acting force is removed, e.g. a string breaks so no inward force is applied .

be something that's producing a force on it. For example, in the case of a child whirling a ball on the end of a string, the child produces the inward force by pulling on the string.

QUESTION 2.6

What is the force responsible for keeping the Earth in its (almost) circular orbit around the Sun?

2.1.5 Forces between objects

An object can only experience a force if is interacting with some other object. *All* forces are the result of pairs of objects interacting in some way and exerting forces on one another. These pairs of forces are described by **Newton's third law of motion**.

> When two bodies interact, they exert forces on one another that are equal in magnitude and opposite in direction.

If the two interacting objects are similar in mass, then both forces can have a noticeable effect. For example, if one ice-skater pushes against another, then they will both accelerate away from one another. But if the objects are very unequal in mass, then the two resulting accelerations will be very different. For example, you are attracted towards the Earth by a gravitational force. The Earth exerts a force on you and you can experience its result: if you jump off a chair you accelerate downwards, in the direction of the force. And as described by Newton's third law, you exert a force of equal magnitude on the Earth. When you jump off a chair you accelerate towards the Earth and the Earth accelerates towards you. But the Earth's mass is so very much greater than yours that its acceleration is absolutely tiny compared with yours, and is not noticeable.

2.2 Gravitational force

2.2.1 Gravitational forces between masses

Perhaps the most important force in astronomy and planetary science is that of **gravitation**. All objects, no matter how small or large their mass, attract each other by a gravitational force. **Newton's law of gravitation**, which describes this attraction, was deduced around 1666.

Two particles, of masses m_1 and m_2 and separated by a distance r, attract each other with a gravitational force whose magnitude F_g is *proportional* to the product of their masses and *inversely proportional* to their separation. See Figure 2.7.

In symbols:

$$F_g = \frac{Gm_1m_2}{r^2} \tag{2.4}$$

where G is the gravitational constant ($G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$, notice its SI units). This law has some intuitively reasonable features. If either of the masses is increased, then F_g increases, and if the distance is increased then F_g decreases.

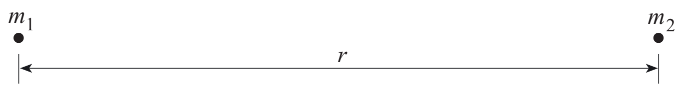


Figure 2.7 Two bodies separated by a distance r.

The separation r is the distance between the *centres* of the two objects, as shown in Figure 2.8. (Strictly, r is the distance between their centres of gravity, but for a spherical object whose mass is distributed symmetrically, such as a star or planet, the centre of gravity *is* the geometric centre.)

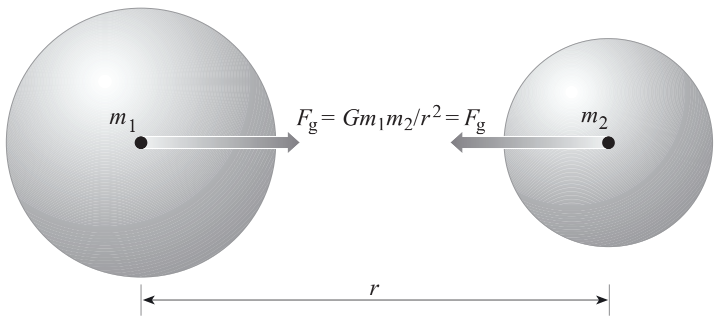


Figure 2.8 The gravitational force F_g between two spherically symmetric objects.

The gravitational force is generally noticeable only when at least one of the objects has a very large mass, as the following example and question show. But because the force is always one of attraction, gravitational forces between objects can never cancel each other out, unlike *electrical forces*, and over large distances the effects of gravity dominate over all other forces.

EXAMPLE 2.3

What is the magnitude of the gravitational force of attraction between you and the Earth when you are on the Earth's surface? The mass of the Earth is 5.97×10^{24} kg and its radius is 6.37×10^{6} m.

From Equation 2.4, if your mass is 60 kg, then

$$F_g = \frac{6.67\times10^{-11}\ \text{N m}^2\ \text{kg}^{-2} \times 60\ \text{kg} \times 5.97\times10^{24}\ \text{kg}}{(6.37\times10^{6}\ \text{m})^2} = 589\ \text{N}$$

QUESTION 2.7

What is the magnitude of the gravitational force between the Earth and Moon? The Moon's mass is 7.35×10^{22} kg and the distance between the centres of the Earth and Moon is 3.85×10^{8} m.

2.2.2 Gravitational field, acceleration and weight

A mass m_1 that experiences a gravitational force due to its interaction with another mass m_2 is said to be in the gravitational field of m_2. A gravitational field can be quantified: it is defined as the gravitational force exerted on unit mass. In symbols, using g to represent the field,

$$g = \frac{F_g}{m_1} \qquad (2.5)$$

and so, from Equation 2.4

$$g = \frac{Gm_2}{r^2} \qquad (2.6)$$

Like force, gravitational field has direction (towards m_2) as well as magnitude. The magnitude is often referred to as the **gravitational field strength**. From Equation 2.5 we can see that g has SI units of N kg^{-1}. The field strength thus depends on the mass m_2 and the square of the distance, r, from its centre.
The force F_g that an object m experiences due to a gravitational field is called its **weight**. So from Equation 2.5 we have

$$F_g = mg \qquad (2.7)$$

EXAMPLE 2.4

Using information from Example 2.3, write down your weight when you are on the Earth's surface.

No calculation is necessary. The force calculated in Example 2.3 *is* your weight. So if your mass is 60 kg then your weight is 589 N.

EXAMPLE 2.5

Using information from Example 2.4, what is the Earth's surface gravity, i.e. the strength of the gravitational field at its surface?

From Equation 2.7

$$g = \frac{F_g}{m_1} = \frac{589\ \text{N}}{60\ \text{kg}} = 9.8\ \text{N kg}^{-1}$$

(Note that the result is the same irrespective of the mass.)

When an object is in a gravitational field, and is not experiencing any forces other than its own weight, it will accelerate in the direction of the field. Its **acceleration due to gravity** (also called the acceleration of free fall) is found by rearranging Equation 2.3, which yields something remarkably similar to Equation 2.5. Using g to represent this gravitational acceleration we can see that it is really the same as field strength, the only (subtle) distinction being that the gravitational field is there all the time, whereas acceleration occurs only when something is falling freely under the influence of gravity.

From Example 2.2 and related text, we can see that the SI units of gravitational field strength ($\mathrm{N\,kg^{-1}}$) are exactly equivalent to those of acceleration ($\mathrm{m\,s^{-2}}$) because $1\,\mathrm{N} = 1\,\mathrm{kg\,m\,s^{-2}}$. So we can say both that the gravitational field at the Earth's surface has magnitude $9.8\,\mathrm{N\,kg^{-1}}$ and that the acceleration due to gravity at the Earth's surface is $9.8\,\mathrm{m\,s^{-2}}$.

QUESTION 2.8

The surface gravity of Mars is $3.7\,\mathrm{N\,kg^{-1}}$. What is the acceleration due to gravity on the surface of Mars? What would be the weight of an astronaut, mass 80 kg, standing on the Martian surface? What is the weight of the same astronaut on the Earth's surface, where $g = 9.8\,\mathrm{N\,kg^{-1}}$?

QUESTION 2.9

On the Moon, an astronaut of mass 75 kg has weight 120 N. What is the strength of the Moon's gravitational field at its surface? The astronaut drops a hammer. What is the hammer's acceleration as it falls?

2.3 Electrical force

If two objects carry an electric **charge**, they will interact by exerting a force on each other which is usually much stronger than their gravitational attraction: the **electrical force**. In SI units, electric charge is measured in coulombs (C), named after the French scientist Charles de Coulomb (1736–1806). If one object has a charge Q_1 and another has a charge Q_2, then the magnitude of the electrical force F_e between them is given by **Coulomb's law**:

$$F_e = \frac{-k_e Q_1 Q_2}{r^2} \tag{2.8}$$

where k_e is a constant that depends on the medium between the charges and r is the distance between them. The sign of Q_1Q_2 determines the direction of the force. If both charges have the same sign (both positive or both negative, so that Q_1Q_2 is positive and F_e negative) then the force is repulsive, whereas if they are of opposite sign (Q_1Q_2 negative, F_e positive) then the force is attractive.

Equation 2.8 applies to point charges, or to two spherically symmetric charge distributions with r being the distance between their centres. In a vacuum, the constant k_e is written, for historical reasons, as $1/4\pi\varepsilon_0$, where ε_0 is called the permittivity of free space. $1/4\pi\varepsilon_0$ has the value $8.99 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$ and ε is the Greek letter epsilon.

Coulomb's law is similar in many ways to Newton's law of gravitation. It depends on the charges of the two objects and on their separation, and in an analogous way to gravitational field we can define the electric field due to any electric charge Q: it is the electrical force per unit charge.

There are, however, some big differences between the two laws. First, whereas mass can only be positive, electric charge can be positive or negative. The second big difference is that, whereas all objects exert gravitational forces on each other, many have zero electric charge — they are electrically neutral — and therefore they experience no electrical force: if either Q_1 or Q_2 in Equation 2.8 is zero, then F_e is also zero. Another big difference is that the electric force can be reduced by placing suitable material between the two charges whereas gravitational force is unaffected by any intervening material. And, not least, the constant k_e is very much greater than G, so between two charged objects reasonably close to each other, the magnitude of the electric force almost always far outweighs that of the gravitational force.

EXAMPLE 2.6

Calculate the magnitude of the electrical force between an electron (charge $Q_1 = -1.60 \times 10^{-19}$ C) and a proton (charge $Q_2 = +1.60 \times 10^{-19}$ C) in a hydrogen atom where they are separated by a distance 5.29×10^{-11} m. Is the force attractive or repulsive?

From Equation 2.8

$$
\begin{aligned}
F_e &= \frac{-k_e Q_1 Q_2}{r^2} \\
&= \frac{-(8.99\times10^{9}\ \mathrm{N\,m^2\,C^{-2}})\times(-1.60\times10^{-19}\ \mathrm{C}\times1.60\times10^{-19}\ \mathrm{C})}{(5.29\times10^{-11}\ \mathrm{m})^2} \\
&= 8.22\times10^{-8}\ \mathrm{N}
\end{aligned}
$$

The force is attractive, because Q_1 and Q_2 have opposite sign, making F_e positive.

QUESTION 2.10

What is the magnitude and direction of the force between two protons, each carrying a charge of $+1.60 \times 10^{-19}$ C and separated by a distance 1.0×10^{-12} m? What would be the force between two helium nuclei, each with a charge twice that of a proton, separated by the same distance?

2.4 Energy

2.4.1 Energy conservation

One of the most important and widespread concepts in science and technology is that of **energy**. The scientific concept is, in essence, the same as the everyday concept — energy is a measure of the capacity of a body to make things happen.

Energy takes many forms. For example, an object has energy by virtue of its motion: a cannon-ball in flight clearly has the capacity to make things happen when it hits something. On the other hand, a cannon-ball held above the ground also has the capacity to make things happen if it is allowed to fall — it has energy by virtue

of its position. A compressed spring, too, has energy, as does a mixture of gunpowder and oxygen, and the food we ingest. Stars rely on energy stored in atomic nuclei to sustain their shining. Any form of stored energy (e.g. in a raised object, in a stretched spring, in an atomic nucleus) is known as **potential energy** as it has the *potential* to make something happen at some time in the future.

When energy is transferred it might cause motion, or maybe sound and/or light is produced as in the case of a cannon-ball hitting the floor or gunpowder igniting. Most often there is also some heating — heating, light and sound are all examples of energy in transit.

Energy is an extraordinarily useful concept for two main reasons. First, practically all processes involve the transfer of energy between different locations or between different objects, and/or its conversion into different forms. Second, all forms of energy can be quantified, and energy transfers and conversions all take place according to a strict system of natural accountancy, summarized as the law of **conservation of energy**: after all the changes have taken place, you always end up with *exactly* the same amount of energy as you started with.

2.4.2 Energy and work

Imagine pushing a supermarket trolley with a constant *force* along level ground so that it gradually picks up speed. You have transferred some energy to the trolley (energy that was previously stored in your body from the food you have eaten) by doing **work**. In the scientific sense, the amount of work you do, W, is defined as the magnitude of the force F that you exert multiplied by the distance, d, that you move in the direction of the force:

$$W = Fd \tag{2.9}$$

and the amount of energy transferred is equal to the amount of work done. Equation 2.9 defines the SI units of energy and work. One joule (J), is the energy transferred when a force of 1 N moves something through 1 m in the direction of the force, so

$$1\,\mathrm{J} = 1\,\mathrm{N\,m} = 1\,\mathrm{kg\,m^2\,s^{-2}}$$

The energy that you transfer by doing work on the trolley may take various forms. If there is no resistance to your pushing force (no friction in the trolley wheels) then the energy of the trolley's motion will be equal to the work you have done. But if there is friction in the wheels, some of the energy you transfer will take other forms (you might hear the wheels squeaking, and the wheel bearings will get warm) and the trolley will not move so fast.

EXAMPLE 2.7

Suppose you push a broken-down car through 15 m with a force of 415 N. How much work do you do?

Using Equation 2.9, $W = 415\,\mathrm{N} \times 15\,\mathrm{m} = 6.2 \times 10^3\,\mathrm{J}$.

QUESTION 2.11

The rocket motors of a spacecraft are used to exert a force of 2.40×10^5 N and accelerate the spacecraft through a distance of 600 m. How much energy is transferred during this operation?

2.4.3 Kinetic energy

Starting from the equivalence of work and energy, and using Equation 2.9, it is possible to derive an expression for an object's **kinetic energy**, i.e. its energy of motion. An object of mass m moving at speed v has kinetic energy E_k where

$$E_k = \frac{1}{2}mv^2 \tag{2.10}$$

EXAMPLE 2.8

A golf ball of mass 5.0×10^{-2} kg moves at $80\,\mathrm{m\,s^{-1}}$. What is its kinetic energy?

From Equation 2.10,

$$E_k = \frac{5.0\times10^{-2}\ \mathrm{kg}\times(80\,\mathrm{m\,s^{-1}})^2}{2}$$
$$= 1.6\times10^2\ \mathrm{J}.$$

EXAMPLE 2.9

A rock, mass 4.0 kg, is ejected from a volcano with initial kinetic energy 800 J. What is its initial speed?

Rearranging Equation 2.10

$$v^2 = \frac{2E_k}{m}$$

so $$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2\times800\ \mathrm{J}}{4.0\ \mathrm{kg}}} = 20\,\mathrm{m\,s^{-1}}$$

QUESTION 2.12

A certain meteoroid (small rocky body) of mass 8.0 kg travels through space at $4.0 \times 10^4\,\mathrm{m\,s^{-1}}$. What is its kinetic energy?

QUESTION 2.13

How fast would an athlete, mass 60 kg, need to sprint in order to have 3000 J of kinetic energy?

2.4.4 Gravitational energy

An the object's potential energy can be increased by lifting it up. If the object is raised though a height h, by a force equal in magnitude to its weight mg, then the work done (this time against gravity) is again mgh and the potential energy is increased by this amount.

$$W = Fd = mgh \tag{2.11}$$

The *potential energy* that an object has, by virtue of its position in a *gravitational field,* is known as gravitational potential energy or **gravitational energy** and is symbolized E_g. Since only the *change* in height is important, rather than the actual height measured from some reference ground level, we usually write

$$\Delta E_g = mg\,\Delta h \tag{2.12}$$

where ΔE_g is the change in gravitational potential energy and Δh the change in height. Notice that the **delta symbol**, Δ, means 'a change in' and is *not* a number multiplying E_g or h.

EXAMPLE 2.10

Calculate the gravitational potential energy transferred to a suitcase, mass 12 kg, when it is lifted up to a luggage rack 2.0 m above the floor, in a gravitational field $9.8\,\mathrm{N\,kg^{-1}}$.

From Equation 2.12, $\Delta E_g = 12\,\mathrm{kg} \times 9.8\,\mathrm{N\,kg^{-1}} \times 2.0\,\mathrm{m} = 240\,\mathrm{J}$. (The answer is better written as $2.4 \times 10^2\,\mathrm{J}$ or 0.24 kJ, as there are only 2 *significant figures*.)

2.4.5 Motion under gravity

If an object falls freely, influenced only by gravitational force (e.g. Newton's apple), the force of gravity does work on it — the object accelerates in the direction of the force. The object gathers speed, in other words its kinetic energy increases. The kinetic energy gained by the falling object is equal to the work done on it by the force of gravity. Put another way, the falling object gains kinetic energy while losing an equal amount of the gravitational energy that it had by virtue of its height, and the total amount of energy remains unchanged as required by the law of conservation of energy.

Provided there is no transfer in other ways such as heating, the kinetic energy gained is *exactly* equal to the potential energy lost, as required by the law of conservation of energy. The overall change in energy must be zero so we can write

$$\Delta(E_g + E_k) = 0$$

or

$$\Delta E_k = -\Delta E_g \tag{2.13}$$

The minus sign simply means that if E_g decreases (ΔE_g is negative) then E_k increases (ΔE_k is positive) and vice versa. The following example shows how to use Equation 2.13 in a calculation.

EXAMPLE 2.11

An apple of mass 0.10 kg drops from a branch at height of 2.4 m and falls freely in a gravitational field of $10\,\mathrm{N\,kg^{-1}}$. What is its kinetic energy just before it hits the ground? How fast will it be travelling?

Its gravitational energy decreases: $\Delta E_g = -mg\,\Delta h$ (Equation 2.12) and so from Equation 2.13

$$\Delta E_k = mg\,\Delta h = 0.10\,\mathrm{kg} \times 10\,\mathrm{N\,kg^{-1}} \times 2.4\,\mathrm{m} = 2.4\,\mathrm{J}$$

Following the same method as Example 2.9 and rearranging Equation 2.10 to make v the subject:

$$v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times 2.4\,\mathrm{J}}{0.10\,\mathrm{kg}}} = 6.9\,\mathrm{m\,s^{-1}}$$

As it deals entirely with changes, not absolute amounts of energy, Equation 2.13 can include situations where the initial kinetic energy is not zero.

EXAMPLE 2.12

Suppose the apple in Example 2.11 did not drop but was thrown vertically downwards with an initial velocity of 6.0 m s^{-1}. How fast will it now be moving just before it hits the ground?

$$\text{Initial } E_\mathrm{k} = \frac{1}{2}mv^2 = \frac{0.10\,\mathrm{kg} \times (6\,\mathrm{m\,s^{-1}})^2}{2} = 1.8\,\mathrm{J}$$

From Equation 2.13 and Example 2.11, its kinetic energy still increases by an amount $\Delta E_\mathrm{k} = mg\,\Delta h = 2.4$ J, so just before it hits the ground

$$\text{Final } E_\mathrm{k} = 2.4\,\mathrm{J} + 1.8\,\mathrm{J} = 4.2\,\mathrm{J}$$

and $$v = \sqrt{\frac{2E_\mathrm{k}}{m}} = \sqrt{\frac{2 \times 4.2\,\mathrm{J}}{0.10\,\mathrm{kg}}} = 9.2\,\mathrm{m\,s^{-1}}$$

QUESTION 2.14

Part of a space probe, mass 50 kg, is plummeting out of control towards the surface of Mars. At a height of 1.0×10^4 m above the surface is it falling vertically at 200 m s^{-1}. The gravitational field is 3.7 N kg^{-1}. What is its kinetic energy just before it hits the surface? How fast is it travelling?

2.4.6 Heat and internal energy

Many energy conversions and transfers lead to a rise in temperature. Sometimes this is deliberate, as when you burn fuel to heat food or your home, and sometimes it is a by-product of some other change. For example, if you ride a bicycle the desired outcome is to use the energy stored in your body to increase your and the bicycle's kinetic energy. At the same time, friction in the bicycle's moving parts causes them to get warm, you get warm, and the road and surrounding air also become slightly warmer. If you hammer a nail, the nail and the hammer can become noticeably warm.

All temperature rises are associated with an increase in something's **internal energy**. All substances are made up of particles (atoms and molecules) that are moving randomly, and the kinetic energy associated with this random motion is one component of an object's internal energy. If we raise the temperature, we increase the kinetic energy of this random motion. The other component of internal energy is the particles' potential energy, i.e. stored energy associated with the forces between the particles. Just as increasing the separation between an object and the Earth increases their gravitational energy, so increasing the separation of particles interacting by the electric force also increases their potential energy. The internal energy is the sum of the kinetic and potential energies of all the particles.

The internal energy of an object can be increased either by doing *work* on it or supplying **heat** or by some combination of the two. Heat is energy that flows from a higher to a lower temperature because of the temperature difference, and when heat is transferred to an object the internal energy of that object increases.

In general, increasing an object's internal energy results in a temperature rise, the only exceptions being if melting or vaporization is involved. Likewise, decreasing

the internal energy generally leads to a fall in temperature. The temperature change ΔT is related to the change in internal energy (usually denoted by Δq), the mass m of the object and its **specific heat** c (also called the specific heat capacity). The specific heat is a property of the *substance* of which the object is made, and it is the change in internal energy required to bring about a temperature change of 1°C (or 1 K, measured on the absolute temperature scale) in 1 kg of the substance. It has SI units J kg^{-1} °C^{-1} or, equivalently, J kg^{-1} K^{-1}. Now ΔT is related to Δq by the equation

$$\Delta q = mc\,\Delta T \tag{2.14}$$

EXAMPLE 2.13

The specific heat of copper is 3.8×10^2 J kg^{-1} °C^{-1}. How much energy is required to heat a copper pan, mass 0.50 kg, from 20 °C to 100 °C? How much energy is required to heat a copper pan of mass 0.25 kg through the same temperature range? How much energy is given out in each case when the pan cools from 100 °C to 20 °C?

In each case ΔT = 80 °C. For the 0.50 kg copper pan, using Equation 2.14:

$$\begin{aligned}\Delta q &= 0.50\,\text{kg} \times 3.8 \times 10^2\,\text{J kg}^{-1}\,°\text{C}^{-1} \times 80\,°\text{C}\\ &= 1.5 \times 10^4\,\text{J}.\end{aligned}$$

This same amount of energy is given out when the pan cools.

If the mass is halved (0.25 kg copper) then Δq will also be halved: $\Delta q = 7.6 \times 10^3$ J (for both heating and cooling).

EXAMPLE 2.14

The specific heat of aluminium is 9.0×10^2 J kg^{-1} °C^{-1}. Will the energy required to heat an aluminium pan through the same temperature range be greater or less than that required to heat a copper pan of the same mass?

The specific heat capacity of aluminium is greater than that of copper, so more energy will be required to heat an aluminium pan of the same mass through the same temperature range.

QUESTION 2.15

Basalt rock has a specific heat capacity of about 1.2×10^3 J kg °C^{-1} and a melting temperature of about 1070 °C. How much energy is required to bring 1.0×10^3 kg of this rock from room temperature (about 20 °C) to its melting temperature? Compare this with the energy required to bring the same mass of water-ice, specific heat 2.1×10^3 J kg^{-1} °C^{-1}, from −100 °C to its melting temperature of 0 °C.

2.4.7 Changes between solid, liquid and gas states

When a solid at its melting temperature changes to a liquid at the same temperature, the particles (atoms and molecules) must acquire enough energy to move around freely rather than being fixed in position. Likewise, when a liquid at its boiling temperature becomes a *gas* at the same temperature, the particles must acquire enough energy to increase their potential energy by becoming much more

widely separated. In either case, there must be an input of energy. Conversely, when a liquid solidifies, or a gas condenses, with no change of temperature, the particles must lose energy. The energy that is transferred to or from a substance as it changes state (or phase) without changing temperature is called **latent heat**. Latent means hidden — the energy change is 'hidden' because it is not associated with a temperature change.

The energy change Δq associated with a particular change of state depends on the substance involved and the mass Δm that changes state. In symbols:

$$\Delta q = L\,\Delta m \qquad (2.15)$$

where L is the **latent heat of vaporization** of the substance, or the **latent heat of melting** (also called the **latent heat of fusion**, as appropriate). In either case, the SI units of L are $\mathrm{J\,kg^{-1}}$.

EXAMPLE 2.15

The latent heat of melting of water-ice is $3.34 \times 10^5\ \mathrm{J\,kg^{-1}}$. How much energy is required to melt 200 g (0.200 kg) of ice at its melting temperature (0 °C)?

From Equation 2.15, $\Delta q = 0.200\ \mathrm{kg} \times 3.34 \times 10^5\ \mathrm{J\,kg^{-1}} = 6.68 \times 10^4\ \mathrm{J}$.

EXAMPLE 2.16

Liquid oxygen boils at –183 °C at normal atmospheric pressure, and its latent heat of vaporization is $2.5 \times 10^5\ \mathrm{J\,kg^{-1}}$. It can be stored in a thermos flask for short periods but heat from warmer surroundings causes it to boil. If 9.5×10^4 J is transferred to a flask of liquid oxygen, what mass of oxygen is converted from liquid to gas?

Rearranging Equation 2.15:

$$\begin{aligned}\Delta m &= \frac{\Delta q}{L} \\ &= \frac{9.5 \times 10^4\ \mathrm{J}}{2.5 \times 10^5\ \mathrm{J\,kg^{-1}}} \\ &= 0.38\ \mathrm{kg}\end{aligned}$$

QUESTION 2.16

Basalt rock's latent heat of melting is about $4.8 \times 10^5\ \mathrm{J\,kg^{-1}}$. How much energy is given out when 1.0×10^3 kg of molten *basalt* from a volcano solidifies at its melting temperature (about 1070 °C)?

QUESTION 2.17

The latent heat of vaporization of water is $2.6 \times 10^6\ \mathrm{J\,kg^{-1}}$. How much water at its boiling temperature (100 °C) could be vaporized by an energy input of 5.2×10^8 J?

2.4.8 Mass and energy

Stars rely on nuclear fusion to generate their vast outputs of energy. One of the major sources of internal heating in planetary bodies (such as the Earth) is radioactive decay. Both these processes involve nuclear reactions in which the total kinetic energy of the resulting particles is much greater than that of the reactants,

and in many cases high-energy photons are also emitted. Vast quantities of energy are produced and the total mass of the products of the reaction is slightly less than that of the reacting particles. This is consistent with the law of conservation of energy, because energy and mass are not distinct quantities but can be converted into one another. In a sense, mass is a form of energy, even though we normally measure mass in quite a different way from energy and the two quantities have different dimensions. The **mass–energy equivalence** between a change in mass Δm and the corresponding change in energy ΔE is described by an equation that arises from Einstein's theory of special relativity:

$$\Delta E = c^2 \, \Delta m \tag{2.17a}$$

where c is the speed of light in a vacuum, $3.00 \times 10^8 \, \mathrm{m\,s^{-1}}$. The energy that an object has solely by virtue of its mass, as described by Equation 2.17a, is called its **rest energy** E_0 (it has this energy even when at rest, i.e. no kinetic energy)

$$E_0 = mc^2 \tag{2.17b}$$

The huge size of the factor c^2 ($= 9.00 \times 10^{16} \, \mathrm{m^2\,s^{-2}}$) means that a tiny amount of mass is equivalent to an enormous amount of energy. In a nuclear reaction less than 1% of the reacting particles' rest energy is converted into other forms but that still produces a vast output.

Reactions between subatomic particles can sometimes lead to the complete annihilation of matter to produce high-energy photons of electromagnetic radiation. In the reverse situation, matter can be produced purely from radiation.

EXAMPLE 2.17

What is the rest energy of 1.00 kg of matter? That is, if *all* the energy in 1.00 kg of matter could be converted into other forms, how much energy would that be?

Putting $m = 1.00$ kg in Equation 2.17b,

$$E_0 = 1.00 \, \mathrm{kg} \times (3.00 \times 10^8 \, \mathrm{m\,s^{-1}})^2 = 9.00 \times 10^{16} \, \mathrm{J}$$

EXAMPLE 2.18

A single electron has mass 9.11×10^{-31} kg. What is its rest energy? If an electron and its antiparticle (a positron) annihilate one another, they produce two photons of equal energy. Given that a positron has exactly the same mass as an electron, what is the energy of each of the photons produced?

Using Equation 2.17b,

$$E_0 = 9.11 \times 10^{-31} \, \mathrm{kg} \times (3.00 \times 10^8 \, \mathrm{m\,s^{-1}})^2 = 8.20 \times 10^{-14} \, \mathrm{J}$$

The total energy in the annihilation is $2 \times 8.20 \times 10^{-14}$ J so the energy of each photon is 8.20×10^{-14} J.

One consequence of this equivalence between mass and energy is that, whenever an object's energy is increased (for example, by heating it or by setting it in motion) its mass also increases. However, the large size of the factor c^2 in Equation 2.17 means that in most situations the change in mass accompanying a change in energy is not noticeable. The change of mass is, however, noticeable in nuclear reactions, or when particles are accelerated to speeds close to that of light so that their kinetic energy is comparable to their rest energy.

QUESTION 2.18

In the Sun's core, hydrogen is converted into helium and about 0.7% of the mass of hydrogen is converted into other forms of energy. How much energy is produced from 1.0 kg of hydrogen?

2.4.9 Non-SI units of energy

When dealing with energies of individual particles, the quantities can be very small, as Example 2.18 showed. The **electronvolt** is a convenient non-SI unit for expressing very small energies.

$$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J} \qquad (2.18a)$$

and $$1\ \text{J} = 1\ \text{eV}/1.60 \times 10^{-19} = 6.25 \times 10^{18}\ \text{eV} \qquad (2.18b)$$

Although it is a non-SI unit, multiples of eV are written using the standard SI prefixes as the following example illustrates.

EXAMPLE 2.19

The rest energy of an electron is 8.20×10^{-14} J (from Example 2.18). Express this in eV and in MeV ($1\ \text{MeV} = 1 \times 10^{6}\ \text{eV}$).

$$E_0 = (8.20 \times 10^{-14} / 1.60 \times 10^{-19})\text{eV} = 5.12 \times 10^{5}\ \text{eV} \approx 0.5\ \text{MeV}$$

QUESTION 2.19

A hydrogen atom at room temperature has kinetic energy 6.21×10^{-21} J.
What is that in eV?

2.4.10 Power

It is often useful to know the rate at which *energy* is converted or transferred. This is known as **power**. The power, P, can be calculated by dividing the energy converted, ΔE, by the time Δt taken to do the conversion:

$$P = \frac{\Delta E}{\Delta t} \qquad (2.19)$$

Power has SI units of joules per second (J s^{-1}), or watts W.

$$1\ \text{W} = 1\ \text{J s}^{-1}$$

EXAMPLE 2.20

Suppose you take 30 seconds to do 6.2×10^{3} J of work (e.g. pushing a car as in Example 2.7). What is your power output?

From Equation 2.19, $P = 6.2 \times 10^{3}\ \text{J}/30\ \text{s} = 2.0 \times 10^{2}\ \text{W}$.

EXAMPLE 2.21

The Sun has a power output (i.e. luminosity) of 3.83×10^{26} W. How much energy does it emit in one year (3.16×10^{7} s)?

Rearranging Equation 2.19:

$$\begin{aligned}\Delta E &= P\,\Delta t\\ &= 3.83 \times 10^{26}\ \text{W} \times 3.16 \times 10^{7}\ \text{s} = 1.21 \times 10^{34}\ \text{J}\end{aligned}$$

QUESTION 2.20

A particular spacecraft manoeuvre (as in Question 2.11) requires an energy conversion of 1.44×10^{8} J. If this takes 30 minutes (1800 s), what is the power of the rocket engines?

QUESTION 2.21

The international space station requires a power input of about 100 kW (1.00×10^{5} W) to run its instruments. How much energy input does it need in one day (8.64×10^{4} s)?

2.5 Answers and comments for Topic 2

QUESTION 2.1

(a) Speeding up, no change of direction. (The dots get further apart. The distance travelled in a given time interval is increasing.) (b) Slowing down, no change of direction. (The dots get closer together. The distance travelled in a given time interval is decreasing.) (c) Changing direction, constant speed. (The path curves, but the dots are all the same distance apart.) (d) Speeding up and changing direction.

QUESTION 2.2

Change in velocity = $1.5\ \text{m s}^{-1} - 0\ \text{m s}^{-1} = 1.5\ \text{m s}^{-1}$

$$\text{acceleration} = 1.5\ \text{m s}^{-1} / 3\ \text{s} = 0.5\ \text{m s}^{-2}$$

QUESTION 2.3

There is no unbalanced force acting. (The downward force of gravity is exactly countered by an upward-acting drag force caused by motion through the air.)

QUESTION 2.4

First calculate the acceleration using change in velocity ÷ time taken:

$$a = 1.5\ \text{m s}^{-1}/10\ \text{s} = 0.15\ \text{m s}^{-2}$$

Then $F = ma = 80\ \text{kg} \times 0.15\ \text{m s}^{-2} = 12\ \text{N}$.

QUESTION 2.5

Rearranging $F = ma$ we get $a = F/m$ and so

$$a = 4.0 \times 10^6\ \text{N}/2.0 \times 10^8\ \text{kg} = 2.0 \times 10^{-2}\ \text{m s}^{-2}$$

QUESTION 2.6

The gravitational attraction between the Sun and the Earth.

QUESTION 2.7

From Equation 2.4 and using the Earth's mass from Example 2.3:

$$\begin{aligned} F_g &= 6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2} \times 7.35 \times 10^{22}\ \text{kg} \times 5.97 \times 10^{24}\ \text{kg}/(3.85 \times 10^8\ \text{m})^2 \\ &= 1.97 \times 10^{20}\ \text{N} \end{aligned}$$

QUESTION 2.8

Acceleration due to gravity is $g_{\text{Mars}} = 3.7\ \text{N kg}^{-1} = 3.7\ \text{m s}^{-2}$.

From Equation 2.7 and using W to represent weight:

$W_{\text{Mars}} = 80\ \text{kg} \times 3.7\ \text{N kg}^{-1} = 296\ \text{N} = 3.0 \times 10^2\ \text{N}$ (two significant figures).

$W_{\text{Earth}} = 80\ \text{kg} \times 9.8\ \text{N kg}^{-1} = 784\ \text{N} = 7.8 \times 10^2\ \text{N}$ (two significant figures).

QUESTION 2.9

From Equations 2.5 and/or 2.7, $g = F_g/m = 120\ \text{N}/75\ \text{kg} = 1.6\ \text{N kg}^{-1}$.

The acceleration due to gravity at the Moon's surface is $1.6\ \text{m s}^{-2}$.

QUESTION 2.10

From Equation 2.8, $F_e = -k_e Q_1 Q_2/r^2$

$$\begin{aligned} F_e &= -\frac{k_e Q_1 Q_2}{r^2} \\ &= \frac{-8.99 \times 10^9\ \text{N m}^2\ \text{C}^{-2} \times (1.60 \times 10^{-19}\ \text{C})^2}{(1.0 \times 10^{-12}\ \text{m})^2} \\ &= -2.3 \times 10^{-4}\ \text{N} \end{aligned}$$

The force is repulsive, because Q_1 and Q_2 both have positive signs, making F_e negative. If the charge of each particle is doubled, then the magnitude of the force is multiplied by 4, i.e. $F_e = -9.2 \times 10^{-4}\ \text{N}$.

QUESTION 2.11

From Equation 2.9, $W = 2.40 \times 10^5\ \text{N} \times 600\ \text{m} = 1.44 \times 10^8\ \text{J}$.

QUESTION 2.12

From Equation 2.10,

$$\begin{aligned} E_k &= 8.0\ \text{kg} \times (4.0 \times 10^4\ \text{m s}^{-1})^2/2 \\ &= 6.4 \times 10^9\ \text{J} \end{aligned}$$

QUESTION 2.13

Rearranging Equation 2.10 as in Example 2.9,

$$v = \sqrt{\frac{2E_\mathrm{k}}{m}} = \sqrt{\frac{2 \times 3000\,\mathrm{J}}{60\,\mathrm{kg}}} = 10\,\mathrm{m\,s^{-1}}$$

QUESTION 2.14

Initial $E_\mathrm{k} = \frac{1}{2}mv^2 = 50\,\mathrm{kg} \times (200\,\mathrm{m\,s^{-1}})^2/2 = 1.0 \times 10^6\,\mathrm{J}$

From Equations 2.12 and 2.13, its kinetic energy increases by an amount

$$\Delta E_\mathrm{k} = mg\,\Delta h$$

$$= 50\,\mathrm{kg} \times 3.7\,\mathrm{N\,kg^{-1}} \times 1.0 \times 10^4\,\mathrm{m} = 1.85 \times 10^6\,\mathrm{J}$$

and so final $E_\mathrm{k} = 2.85 \times 10^6\,\mathrm{J}$.

Using Equation 2.10

$$v = \sqrt{\frac{2E_\mathrm{k}}{m}} = \sqrt{\frac{2 \times 2.85 \times 10^6\,\mathrm{J}}{50\,\mathrm{kg}}} = 336\,\mathrm{m\,s^{-1}}$$

(which should be rounded to $3.4 \times 10^2\,\mathrm{m\,s^{-1}}$ or $0.34\,\mathrm{km\,s^{-1}}$, as there are only 2 significant figures).

QUESTION 2.15

Basalt: $\Delta T = 1050\,°\mathrm{C}$. From Equation 2.14,

$$\Delta q = 1.0 \times 10^3\,\mathrm{kg} \times 1.2 \times 10^3\,\mathrm{J\,kg^{-1}\,°C^{-1}} \times 1050\,°\mathrm{C}$$

$$= 1.3 \times 10^9\,\mathrm{J}$$

Water-ice: $\Delta T = 100\,°\mathrm{C}$,

$$\Delta q = 1.0 \times 10^3\,\mathrm{kg} \times 2.1 \times 10^3\,\mathrm{J\,kg^{-1}\,°C^{-1}} \times 100\,°\mathrm{C}$$

$$= 2.1 \times 10^8\,\mathrm{J}$$

QUESTION 2.16

From Equation 2.15,

$$\Delta q = 4.8 \times 10^5\,\mathrm{J\,kg^{-1}} \times 1.0 \times 10^3\,\mathrm{kg} = 4.8 \times 10^8\,\mathrm{J}$$

QUESTION 2.17

As in Example 2.16, rearranging Equation 2.15 to make Δm the subject:

$$\Delta m = \frac{\Delta q}{L}$$

$$= \frac{5.2 \times 10^8\,\mathrm{J}}{2.6 \times 10^6\,\mathrm{J\,kg^{-1}}}$$

$$= 2.0 \times 10^2\,\mathrm{kg}$$

QUESTION 2.18

Using Equation 2.17 as in Example 2.17, $\Delta m = 0.7 \times 10^{-2}\,\text{kg} = 7 \times 10^{-3}\,\text{kg}$. In Equation 2.17, $\Delta E = 7 \times 10^{-3}\,\text{kg} \times (3.00 \times 10^{8}\,\text{m s}^{-1})^2 = 6.3 \times 10^{14}\,\text{J}$.

QUESTION 2.19

From Equation 2.18b, $E_\text{k} = (6.21 \times 10^{-21}\ /\ 1.60 \times 10^{-19})\text{eV} = 3.88 \times 10^{-2}\,\text{eV}$.

QUESTION 2.20

Using Equation 2.19, $P = 1.44 \times 10^{8}\,\text{J}/1800\,\text{s} = 8.00 \times 10^{4}\,\text{W}$.

QUESTION 2.21

Using Equation 2.19 as in Example 2.21,

$$\Delta E = 1.00 \times 10^{5}\,\text{W} \times 8.64 \times 10^{4}\,\text{s} = 8.64 \times 10^{9}\,\text{J}$$

TOPIC 3
WAVES AND RADIATION

3.1 Waves

A **wave** may be defined as a disturbance that transfers energy from one place to another. Water waves, seismic waves, shock waves, sound and light are all examples of wave motion. All these except light require some material to travel through, while light involves electrical and magnetic disturbances that can travel most easily in a vacuum (empty space). Water waves, light and some seismic waves (e.g. S-waves) are **transverse waves** which means that the displacements are at right angles to the direction in which the waves are travelling. Sound waves, shock waves and some seismic waves (P-waves) are **longitudinal waves**: the disturbances are to and fro along the direction of wave travel, and involve variations in pressure and density (rather like stretching and squashing a spring).

The simplest sorts of waves involve a regular repeating displacement that can be represented as in Figure 3.1. Figure 3.1a shows how the displacement varies with position at one instant in time, while Figure 3.1b shows how the displacement varies with time at one position. The distance between two adjacent wave crests is the **wavelength**, usually represented by the Greek letter lambda, λ, and the time for one complete 'up and down' cycle is the **period**, T. The maximum displacement is the **amplitude**, A.

The number of complete cycles of the wave per unit time is the **frequency**, f. Frequency has SI units of s^{-1} (number of cycles per second) or hertz, Hz. 1 Hz = 1 s^{-1}. Frequency is the reciprocal of the period:

$$f = \frac{1}{T} \tag{3.1}$$

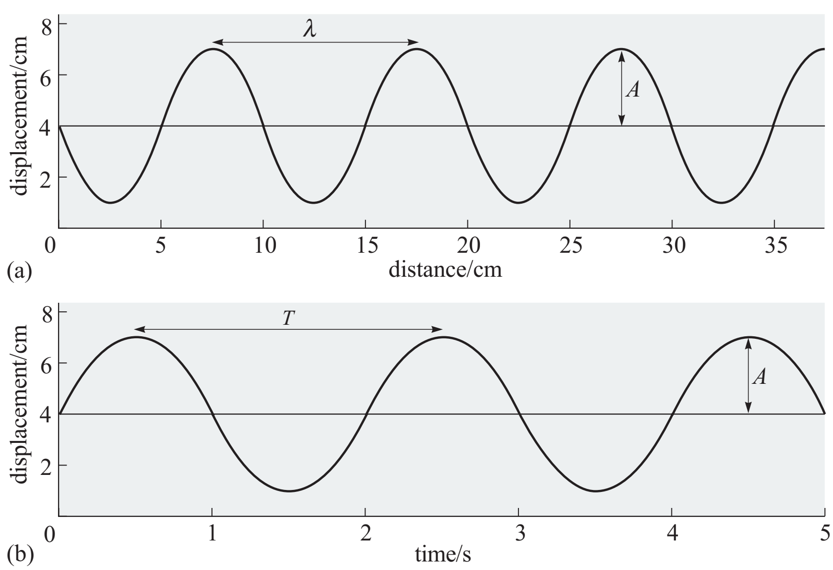


Figure 3.1 A simple wave represented as a graph of (a) displacement against distance and (b) displacement against time.

The wave speed v is the distance the wave travels divided by the time taken to travel. Comparing the two parts of Figure 3.1, the disturbance travels a distance λ in a time interval of T so

$$v = \frac{\lambda}{T}$$

The **wave equation** is more usually written

$$v = f\lambda \qquad (3.2)$$

EXAMPLE 3.1

The musical note, middle C, has a frequency 256 Hz. What is the period of this note?

Sound waves travel in air at about 330 m s^{-1}. If you sing middle C, what is the wavelength of the wave you produce in the air?

Using Equation 3.1

$$T = \frac{1}{256\,\text{Hz}} = 3.9 \times 10^{-3}\ \text{s}$$

Rearranging Equation 3.2,

$$\lambda = \frac{v}{f} = \frac{330\,\text{m}\,\text{s}^{-1}}{256\,\text{Hz}} = 1.30\,\text{m}$$

EXAMPLE 3.2

BBC Radio 4 long-wave broadcasts on a wavelength 1500 m. The controls on some radio sets are labelled with frequency, and Radio 4 is found at 200 kHz (2.00×10^5 Hz). What is the speed of these radio waves in air?

From Equation 3.2, $v = 2.00 \times 10^5\ \text{Hz} \times 1500\ \text{m} = 3.00 \times 10^8\ \text{m}\,\text{s}^{-1}$.

QUESTION 3.1

Seismic P-waves travel through the Earth's crust at about 6.5×10^3 m s^{-1}. If such a wave has a period of 2.0 s, what is its wavelength? What is the frequency of this wave?

3.2 Electromagnetic radiation

Nearly all the information we have about astronomical objects comes from studying the **electromagnetic radiation** we receive from them. This radiation includes light, radio waves and X-rays. When electromagnetic radiation is travelling through space it can be precisely described and understood in terms of *waves* (for example, the wavelength can be measured). All these waves involve electrical and magnetic disturbances that travel fastest in a vacuum, where they have a speed of 3.00×10^8 m s^{-1}. This speed is denoted by c and is often referred to simply as the speed of light. The *wave equation* (Equation 3.2) for electromagnetic radiation is usually written

$$c = f\lambda \qquad (3.3)$$

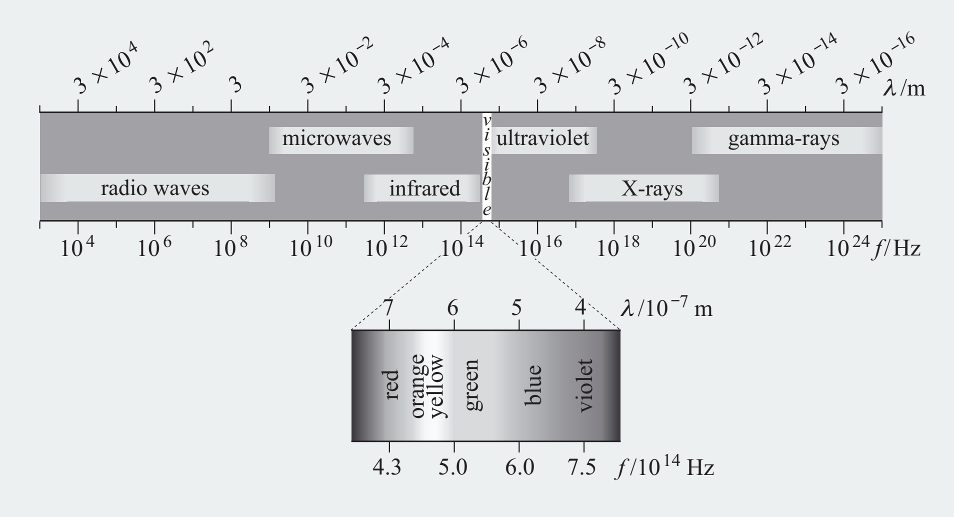


Figure 3.2 The electromagnetic spectrum.

As Figure 3.2 shows, electromagnetic waves have a vast range of frequencies and wavelengths. **Visible radiation** is that which we can see, and covers only a very narrow range of wavelengths. Radiation with wavelengths somewhat longer than those of red light is called **infrared**, and that with wavelengths shorter than violet light is **ultraviolet**. The longest-wavelength, lowest-frequency waves are the **radio** waves, and the radiations with the shortest wavelength and highest frequency are the **X-rays** and **gamma-radiation** (represented by the Greek gamma, γ). Collectively, the full range of waves is known as the **electromagnetic spectrum**. It is the different wavelengths and frequencies that determine the different properties of the waves, for example their ability to penetrate matter, or to cause heating or ionization, or the different ways in which instruments need to be designed to manipulate and detect them. As the range of values is so large, SI prefixes are generally used to express the very small and very large numbers required.

EXAMPLE 3.3

Red light emitted from hot interstellar hydrogen atoms has a wavelength 656 nm, where 1 nm = 1×10^{-9} m. What is the frequency of this radiation?

Wavelength $\lambda = 656 \times 10^{-9}\ \text{m} = 6.56 \times 10^{-7}\ \text{m}$ (see Topic 1). From Equation 3.3,

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}\ \text{m s}^{-1}}{6.56 \times 10^{-7}\ \text{m}} = 4.57 \times 10^{14}\ \text{Hz}$$

QUESTION 3.2

Cold interstellar hydrogen atoms emit radio waves with a frequency 1400 MHz, where 1 MHz = 1×10^{6} Hz. What is their wavelength?

3.3 Photons

When electromagnetic radiation interacts with matter, it can best be understood and described as a stream of particles. A particle of electromagnetic radiation is called a **photon**. Each photon has a distinct energy, and this determines how the photon behaves (for example, whether it can dislodge an electron from an atom to produce an ion).

The relationship between the wave and particle pictures of electromagnetic radiation is given by

$$E_{ph} = hf \tag{3.4}$$

where E_{ph} is the energy of the photon and h is the Planck constant 6.63×10^{-34} J s.

EXAMPLE 3.4

The red light from interstellar hydrogen atoms has a frequency 4.57×10^{14} Hz. What is the energy of one of its photons?

Using Equation 3.4,

$$\begin{aligned} E_{ph} &= 6.63 \times 10^{-34}\ \text{J s} \times 4.57 \times 10^{14}\ \text{Hz} \\ &= 3.03 \times 10^{-19}\ \text{J} \end{aligned}$$

As Example 3.4 shows, single photons have extremely small energy. Rather than expressing such energies in joules, we often use the non-SI unit of energy, the electronvolt (eV), where 1 eV = 1.60×10^{-19} J. However, a warning: for many calculations you need to express energy in SI units, so sometimes you may need to convert from eV to joules.

EXAMPLE 3.5

Express the photon energy of red hydrogen light (Example 3.4) in eV.

$$\begin{aligned} E_{ph} &= 3.03 \times 10^{-19}\ \text{J} \\ &= (3.03 \times 10^{-19}/1.60 \times 10^{-19})\ \text{eV} \\ &= 1.89\ \text{eV} \end{aligned}$$

EXAMPLE 3.6

What is the frequency of electromagnetic radiation that has photons of energy 4.00 keV? Note that 1 keV = 10^3 eV.

As 1 eV = 1.60×10^{-19} J, then

$$\begin{aligned} E_{ph} &= 4.00 \times 10^3\ \text{eV} = (4.00 \times 10^3 \times 1.60 \times 10^{-19})\ \text{J} \\ &= 6.40 \times 10^{-16}\ \text{J} \end{aligned}$$

From Equation 3.4,

$$f = \frac{E_{ph}}{h} = \frac{6.40 \times 10^{-16}\ \text{J}}{6.63 \times 10^{-34}\ \text{J s}} = 9.65 \times 10^{17}\ \text{Hz}$$

QUESTION 3.3

Radio astronomers sometimes use telescopes with receivers tuned to a frequency of 2.7 GHz. Note that 1 GHz = 1×10^9 Hz. What is the energy of a single photon of the radiation they receive? Express your answer in joules and in eV.

QUESTION 3.4

A certain colour of violet light has photons with energy 3.20 eV. What is the frequency of this light?

3.4 Answers and comments for Topic 3

QUESTION 3.1

From Equations 3.1 and 3.2, $\lambda = v/f = vT = 6.5 \times 10^3\ \text{m s}^{-1} \times 2.0\ \text{s} = 1.3 \times 10^4\ \text{m}$.

From Equation 3.1, $f = 1/T = 1/2.0\ \text{s} = 0.50\ \text{Hz}$.

QUESTION 3.2

Frequency $f = 1400 \times 10^6\ \text{Hz} = 1.40 \times 10^9\ \text{Hz}$

From Equation 3.3, $\lambda = c/f = 3.00 \times 10^8\ \text{m s}^{-1}/1.40 \times 10^9\ \text{Hz} = 0.21\ \text{m}$.

QUESTION 3.3

Using Equation 3.4,

$$E_{ph} = hf = 6.63 \times 10^{-34}\ \text{J s} \times 2.7 \times 10^9\ \text{Hz}$$
$$= 1.79 \times 10^{-24}\ \text{J}$$

To convert to units of eV,

$$E_{ph} = (1.79 \times 10^{-24}/1.60 \times 10^{-19})\ \text{eV}$$
$$= 1.12 \times 10^{-5}\ \text{eV}$$

QUESTION 3.4

Following the method of Example 3.6,

$$E_{ph} = 3.20\ \text{eV} = 3.20 \times 1.60 \times 10^{-19)}\ \text{J}$$
$$= 5.12 \times 10^{-19}\ \text{J}$$

From Equation 3.4,

$$f = E_{ph}/h = 5.12 \times 10^{-19}\ \text{J}/6.63 \times 10^{-34}\ \text{J s}$$
$$= 7.72 \times 10^{14}\ \text{Hz}$$

TOPIC 4
MATTER

4.1 States of matter

4.1.1 Arrangements of particles

All matter is made of up tiny particles (electrons, ions, atoms, molecules) that are in perpetual random motion. The large-scale properties of matter, including density, pressure, temperature, state (and others such as 'hardness' or 'runniness') are related to the way these particles are arranged and the way they move.

By the **state of matter** (or **phase of matter**) we mean whether it is a solid, liquid or gas. Any substance can exist in *any* of these states, depending on its temperature and pressure. For example, oxygen and carbon dioxide are gases at normal room temperature and pressure, but if oxygen is cooled to –183 °C it liquefies and if carbon dioxide is cooled by a similar amount it solidifies (to become so-called dry ice). Iron is solid at room temperature and pressure but if heated to 1535 °C it melts to a liquid and at 3027 °C it becomes a gas.

In the **solid** state, the atoms or molecules are packed close together in fixed positions and their only motion is vibration (see Figure 4.1, the spoon). The fixed arrangement of particles means that a solid keeps its shape. In **metals** and some other solids known as semiconductors, electrons are able to move freely throughout the material, enabling them to conduct electricity. If the particles are arranged in regular rows and layers, the solid is then a **crystal**; metals and minerals have a crystalline structure. A single crystal has a regular shape that depends on the arrangement of the particles.

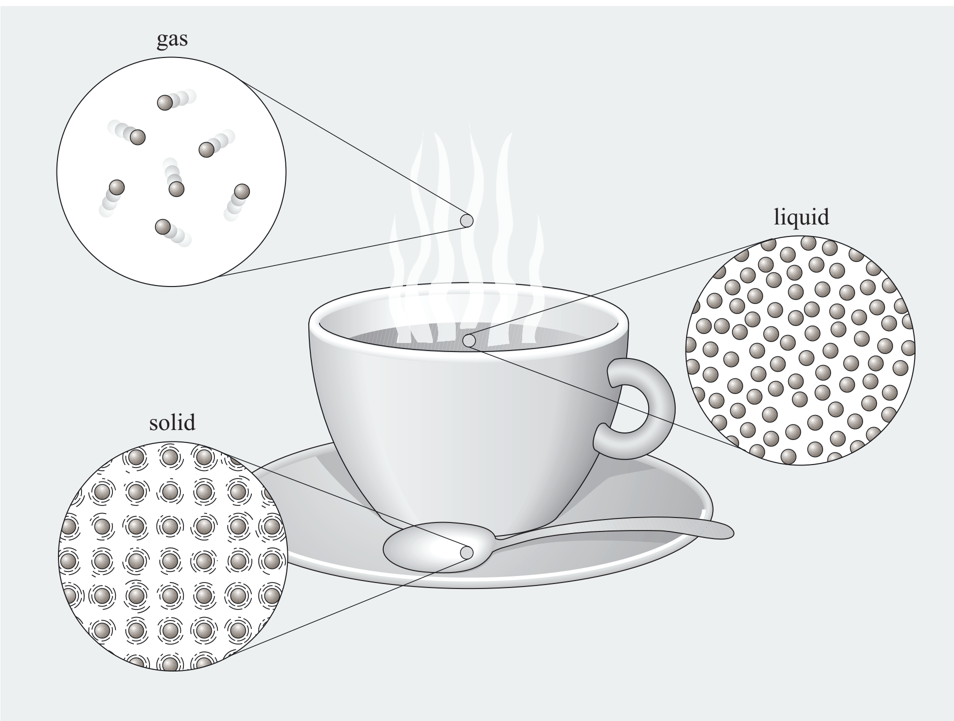


Figure 4.1 Schematic arrangements of particles in a solid (the spoon), a liquid (the tea), and a gas (the 'steam', or vapour).

The particles in **liquid** are also close together — about as close as those in a solid — only now they are not fixed in place but can move around freely (Figure 4.1, the tea) as they have more energy than the particles of a solid. Liquids therefore flow to take up the shape of their container (in this case, a cup) while maintaining a constant volume.

A **gas** is characterized by being made up of atoms or molecules that are widely separated and moving around at high speed (Figure 4.1, the steam), colliding frequently with one another. A gas will therefore flow and expand its volume rapidly to fill any container in which it is placed. Since both liquids and gases can flow freely, they are collectively known as **fluids**.

Finally we come to **plasma**, sometimes called the *fourth* state of matter. A plasma is a gas composed largely of ions and electrons which are free to move independently. As these particles have electric charge, they interact with one another via electrostatic forces, and this considerably affects the properties of the substance. Stars, and many regions of the interstellar medium, consist largely of plasma.

QUESTION 4.1

What is wrong with each of the following statements?

(i) Aluminium is a solid.

(ii) Nitrogen is a gas.

(iii) All gases are plasmas.

4.1.2 Density

Solids, liquids and gases are, loosely speaking, characterized by having different **mass density**, (generally just called the **density**) and this in turn is related to the way their particles are arranged. The density of a substance is its mass per unit volume; in SI units, it is the mass of $1\,\text{m}^3$ of the substance, so the SI units of density are $\text{kg}\,\text{m}^{-3}$. Given the mass m and volume V of a sample, its density, usually denoted by the Greek letter rho, ρ, can be calculated from

$$\rho = \frac{m}{V} \tag{4.1}$$

The densities of the liquid phase and the solid phase of a given substance are similar. On changing between the solid and liquid states the separation of the particles changes very little, so the same mass occupies a very similar volume.

Gases, in general, have much lower densities than solids or liquids, as their particles are much further apart and so a given mass of gas occupies a much larger space than the same mass of that substance when it is in the solid or liquid state. For example, liquid water at its boiling temperature and normal atmospheric pressure has a density of $1 \times 10^3\,\text{kg}\,\text{m}^{-3}$, whereas steam at the same temperature and pressure has a density of only about $0.6\,\text{kg}\,\text{m}^{-3}$.

EXAMPLE 4.1

If the average density of the materials that make up the planet Jupiter is $1.33 \times 10^3\,\mathrm{kg\,m^{-3}}$, and Jupiter's volume is $1.43 \times 10^{24}\,\mathrm{m^3}$, what is Jupiter's mass?

From Equation 4.1

$$m = \rho V$$
$$= 1.33 \times 10^3\,\mathrm{kg\,m^{-3}} \times 1.43 \times 10^{24}\,\mathrm{m^3} = 1.90 \times 10^{27}\,\mathrm{kg}$$

QUESTION 4.2

A certain meteorite has a mass of 2.34 kg and a volume $5.22 \times 10^{-4}\,\mathrm{m^3}$. What is its density?

When we are dealing with the behaviour of individual particles, particularly when densities are very low (for example in the interstellar medium), it is often useful to characterize substances by their **number density**, i.e. the *number* of particles per unit volume. Number density is usually denoted n, and has SI units $\mathrm{m^{-3}}$. A word of warning: sometimes we use number density to mean the total number of particles per unit volume (regardless of their nature) whereas at other times we are interested in the number density of a particular substance, e.g. the number of hydrogen atoms per unit volume — this is usually clear from the context.

Number density and mass density are closely related. If there are n particles per unit volume, and each particle has a mass M, then the overall mass per unit volume is nM, in other words

$$\rho = nM \tag{4.2}$$

EXAMPLE 4.2

In a certain region of the interstellar medium, the number density of hydrogen atoms is $1.00 \times 10^{11}\,\mathrm{m^{-3}}$. Given that the mass of a hydrogen atom is $1.67 \times 10^{-27}\,\mathrm{kg}$, what is the mass density of hydrogen in this region?

From Equation 4.2, $\rho = 1.00 \times 10^{11}\,\mathrm{m^{-3}} \times 1.67 \times 10^{-27}\,\mathrm{kg} = 1.67 \times 10^{-16}\,\mathrm{kg\,m^{-3}}$.

QUESTION 4.3

In the Sun's core, the mass density is about $1 \times 10^5\,\mathrm{kg\,m^{-3}}$. If you assume that the Sun's core is mainly composed of *protons*, each with mass $1.67 \times 10^{-27}\,\mathrm{kg}$, what is the number density of protons in the Sun's core?

4.2 Temperature and pressure

4.2.1 Kinetic energy and absolute temperature

The average kinetic energy of an object's constituent particles is related to its **temperature**. As temperature rises, the average particle kinetic energy increases. Likewise, as an object cools, so the average kinetic energy of its particles decreases. Eventually a point is reached at which the particles can lose no more energy and no further cooling can be achieved. This point is the same for all substances, and the temperature at which this occurs is known as **absolute zero**

— the lowest temperature possible. On the Celsius temperature scale (formerly known as centigrade) this temperature has a value of –273.15 °C.

For scientific purposes it is useful to define a temperature scale that starts at absolute zero. This is the **absolute temperature scale**, also known as the **Kelvin scale** after physicist and engineer William Thomson, Lord Kelvin (1824–1907). The unit of temperature on this scale is the kelvin (K) (not 'degree kelvin', and not °K), and the kelvin is the SI unit of temperature. Figure 4.2 shows the relationship between the Kelvin and Celsius temperature scales. A kelvin is the same size as a °C, so to convert from °C into K you just add 273.15 to the Celsius temperature:

$$(\text{temperature in K}) = (\text{temperature in °C}) + 273.15 \qquad (4.3a)$$

or, in symbols

$$T/\text{K} = T/°\text{C} + 273.15 \qquad (4.3b)$$

There is a direct relationship between the particles' kinetic energy and the absolute temperature. The simplest case is that of a gas made up of single atoms, where the only contribution to the internal energy is the motion of each atom as a whole. The atoms will transfer energy to one another as they collide, so it is not possible to pinpoint the kinetic energy of any particular one, but the total amount of energy that they share between them depends only on the temperature. At an absolute temperature T, the average kinetic energy of each particle, E_k is given by

$$E_k = \frac{3kT}{2} \qquad (4.4)$$

where k is the **Boltzmann constant**, 1.38×10^{-23} J K^{-1}, named after Austrian physicist Ludwig Boltzmann (1844–1906). If a gas is made up of two or more types of atom, then the average kinetic energy per atom is still given by Equation 4.4, but the less massive atoms will on average be moving faster in order to have the same kinetic energy (E_k also equals $mv^2/2$ so if m is small then v must be large to give the same E_k).

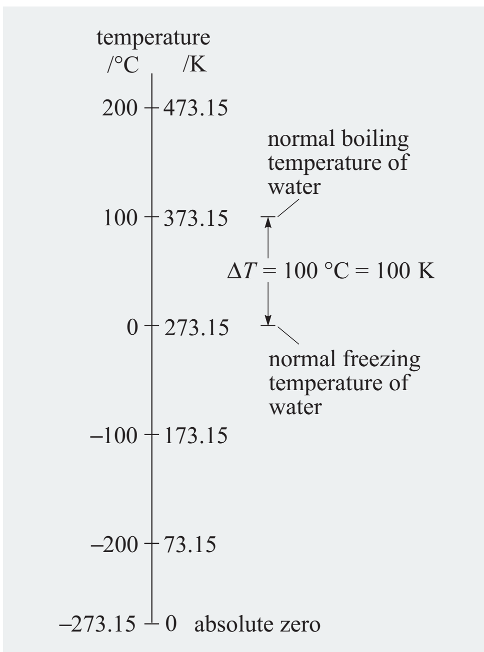


Figure 4.2 The Celsius and Kelvin temperature scales.

In gases with molecules that can rotate and vibrate as well as moving to and fro, there are other contributions to the internal energy but Equation 4.4 still describes the so-called translational kinetic energy, i.e. the kinetic energy of the to-and-fro motion.

For other types of substance (solids for example, or plasmas) the relationship between temperature and internal energy is not quite the same as that shown in Equation 4.4, but there is still a direct connection between temperature and the energy of the particles.

EXAMPLE 4.3

What is the average kinetic energy of an atom in a gas whose temperature is 50 °C?

First express T in SI units of kelvin: from Equation 4.3, $T = 323.15\,\mathrm{K}$

Then from Equation 4.4,

$$E_\mathrm{k} = \frac{3 \times 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}} \times 323.15\,\mathrm{K}}{2} = 6.7 \times 10^{-21}\ \mathrm{J}$$

QUESTION 4.4

The temperature near the top of the Earth's atmosphere is about 1000 K. What is this temperature in °C? What is the average kinetic energy of atoms in the atmosphere here?

QUESTION 4.5

The mass of a nitrogen atom is 14 times that of a hydrogen atom, and that of an oxygen atom is 16 times that of a hydrogen atom. In a gas that is a mixture of hydrogen, oxygen and nitrogen atoms, which atoms will have, on average, the most kinetic energy? Which atoms will, on average, have the highest speeds?

4.2.2 Pressure

Pressure, p, is defined as force per unit area

$$p = \frac{F}{A} \tag{4.5}$$

where A is the area (measured 'square on') over which a force F is acting. The SI unit of pressure is the pascal, Pa, named after the French scientist Blaise Pascal (1623–1662). Note that $1\ \mathrm{Pa} = 1\ \mathrm{N\,m^{-2}}$.

EXAMPLE 4.4

If your weight is 600 N and the area of your shoe soles in contact with the floor is $0.020\ \mathrm{m^2}$, what is the pressure you exert on the floor? If you lift up one foot so that you halve the area in contact with the floor, what happens to the pressure you exert?

$$p = \frac{600\ \mathrm{N}}{0.020\ \mathrm{m^2}} = 3.00 \times 10^4\ \mathrm{Pa}$$

If you halve the area the pressure is doubled to 6.00×10^4 Pa.

The pressure of the atmosphere, which is greatest at sea-level and decreases with height, and the increase of pressure with depth under water, can be explained in terms of the weight of the overlying air and water. However, within a fluid, the random motion of the particles ensures that pressure is exerted equally in all directions and is not associated with any particular direction.

4.2.3 Gas pressure

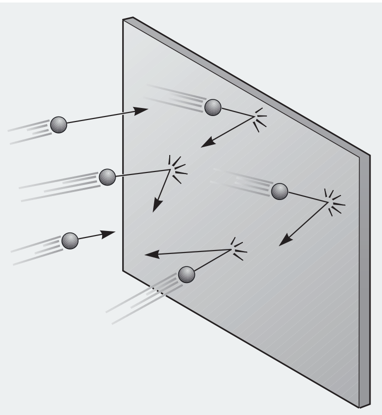


Figure 4.3 A gas exerts a force on a surface as a result of collisions.

The continuous motion of gas molecules gives rise to a pressure as they bombard any surfaces that they strike, as shown schematically in Figure 4.3.

The pressure, i.e. the force on a given area of surface, can be increased in two ways. First, if the number density, n, of gas particles increases then there are more frequent collisions. Second, if the temperature, T, increases then the particles move faster and the collisions not only become more frequent but each collision is more violent so giving rise to a greater force. These relationships can be summarized:

$$p = nkT \tag{4.6}$$

where k is the Boltzmann constant, $1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ and the temperature, T, is the absolute temperature.

EXAMPLE 4.5

The pressure near the top of the Earth's atmosphere is much less than that at sea-level, and yet the temperature is much greater. What can you deduce about the number density of gas particles near the top of the atmosphere?

The number density must be much less than at sea-level. If it were the same, then the higher temperature would give rise to a higher pressure.

EXAMPLE 4.6

At sea-level, the pressure of the Earth's atmosphere is 1.0×10^5 Pa and the number density of gas molecules is about $2.5 \times 10^{25}\,\mathrm{m^{-3}}$. What do these values predict for the temperature at the Earth's surface?

Rearranging Equation 4.6 to make T the subject

$$T = \frac{p}{nk} = \frac{1.0 \times 10^5\ \mathrm{Pa}}{(2.5 \times 10^{25}\ \mathrm{m^{-3}}) \times (1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}})} = 290\,\mathrm{K}\ (= 17\,°\mathrm{C})$$

QUESTION 4.6

At the surface of Venus the particle number density in the atmosphere is about $9.0 \times 10^{26}\,\mathrm{m^{-3}}$ and the temperature is about 733 K. What would you predict for the atmospheric pressure at this location?

4.3 Atoms and their constituents

4.3.1 Atomic structure

Atoms are the basic components of matter. They come in many types but all are very small, about 2×10^{-10} m in diameter. An atom consists of a cloud of **electrons**, each with a negative electric charge, surrounding a tiny positively charged **nucleus**. The nucleus has a diameter of only about 1×10^{-14} m but accounts for nearly all the mass of an atom. In general a nucleus consists of a tightly-bound mixture of **neutrons** and **protons**. The structure of an atom is shown schematically in Figure 4.4. While in some ways, this picture is quite unrealistic (the electrons are not confined to narrow orbits, for example, but rather form a fuzzy cloud of charge around the nucleus) it does represent many key features of atomic structure.

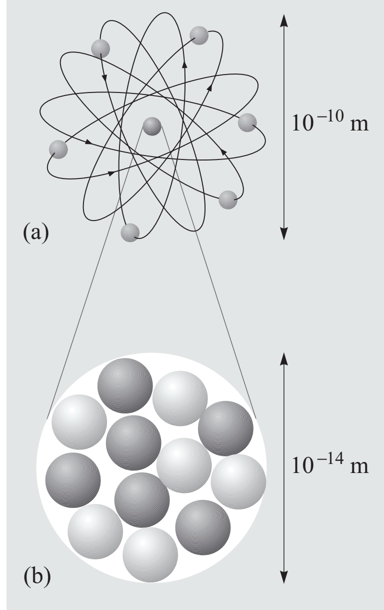


Figure 4.4 A schematic representation of a typical atom: (a) electrons move around the nucleus, (b) the nucleus is made up of protons and neutrons.

The masses and electric charges of the electron, proton and neutron are given in Table 4.1. Notice that the charges of the electron and proton are *exactly* the same size but of opposite sign. An atom contains equal numbers of protons and electrons, so is electrically neutral. However, an atom can gain or lose electrons in which case it is called an **ion**. The process of losing electrons is called **ionization**.

Table 4.1 The particles that make up an atom.

Particle and symbol	mass/kg	charge/C
electron e or e^-	9.109×10^{-31}	-1.602×10^{-19}
proton, p	1.6726×10^{-27}	$+1.602 \times 10^{-19}$
neutron, n	1.6749×10^{-27}	zero

There are many types of atom, each being characterized by the number of protons and neutrons in its nucleus. All the atoms of a particular **chemical element**, represented by its own unique chemical symbol, have the same number of protons in the nucleus. The number of protons is called the **atomic number** and given the symbol Z. The number of protons in turn determines the number of electrons in the atom, and it is the electrons that take part in chemical reactions. It is the electrons that give an atom its particular chemical properties.

Ions are denoted by a chemical symbol with a superscript indicating the net change in terms of the proton's charge. So if an atom of aluminium loses an electron it becomes Al^+, if it loses two electrons it becomes Al^{2+}, and if it gains an electron it is Al^-.

QUESTION 4.7

(i) Write down the chemical symbol for the ion created when an atom of iron loses two electrons. (ii) What must happen to an atom of chlorine, Cl, in order to create the ion Cl^-?

4.3.2 Isotopes

Neutrons and protons are collectively called **nucleons**. The total number of nucleons in an atom is called the **mass number** and given the symbol A. To a good approximation, a proton and a neutron have the same mass, 1.67×10^{-27} kg, and so the mass of a nucleus is very close to A times this mass. As the electron's mass is so very much smaller than that of a nucleon the mass of the nucleus is, for most purposes, the same as the mass of the atom.

Nuclei of any given element, characterized by its **atomic number** Z, can have various numbers of neutrons and hence various different values of mass number A. These different nuclei are all **isotopes** of the same element. Different isotopes of an element will all undergo the same chemical reactions but will have different nuclear reactions.

A particular isotope of a particular element corresponds to a particular type of nucleus, called a **nuclide**, and is specified by its values of A and Z. The usual way of representing a nuclide is ${}^{A}_{Z}\mathrm{X}$, where X is the chemical symbol. For example, one isotope of aluminium is ${}^{27}_{13}\mathrm{Al}$. All isotopes of aluminium have 13 protons, and this particular one also has 14 neutrons giving it a mass number of 27. The isotope is often referred to as aluminium-27 in order to distinguish it from another isotope aluminium-26 which has only 13 neutrons and hence a mass number 26. Quite often we omit the atomic number from the symbol, ${}^{27}\mathrm{Al}$, because it adds no information that is not already implied by the chemical symbol, although when writing nuclear reaction equations it is useful to include it.

The most common isotope of the lightest element, hydrogen, has a nucleus consisting of just a single proton. There are two less-common hydrogen isotopes: **deuterium**, also called 'heavy hydrogen', has a neutron and a proton in the nucleus, and **tritium**, has a proton and two neutrons. These isotopes are unusual in that they are sometimes given their own chemical symbols. Deuterium can be represented as either ${}^{2}_{1}\mathrm{H}$ or ${}^{2}_{1}\mathrm{D}$, while tritium can be either ${}^{3}_{1}\mathrm{H}$ or ${}^{3}_{1}\mathrm{T}$.

QUESTION 4.8

An isotope of iron is represented by ${}^{56}_{26}\mathrm{Fe}$. How many protons are in its nucleus? How many neutrons?

QUESTION 4.9

Barium (Ba) has $Z = 56$. Write down the symbol for the isotope of barium that contains 81 neutrons.

4.3.3 Atomic energy levels

An electron and an atomic nucleus have kinetic energy and potential energy. The potential energy depends on the distance between the electron and the nucleus and arises because a nucleus and an electron have opposite electric charge and so attract one another by the electric force. Just as increasing the separation between an object and the Earth increases their gravitational potential energy, so increasing the separation of particles interacting by the electric force also increases their electric potential energy. The total internal energy of an atom is the sum of the kinetic and potential energies due to the motion and positions of the nucleus and all the electrons.

Within any atom, there can only be certain discrete energies. These permitted **energy levels** are unique for each chemical element and are determined according to the laws of quantum physics that become important when we are dealing with matter on very small scales. The simplest case is that of hydrogen, which has only one electron. Its energy levels are shown in Figure 4.5. They are labelled $n = 1, 2, 3$ … starting with the lowest level, known as the **ground state**. This corresponds to the electron being close to the nucleus. The higher levels correspond to the atom being in an **excited state**, meaning that it has energy over and above its minimum value.

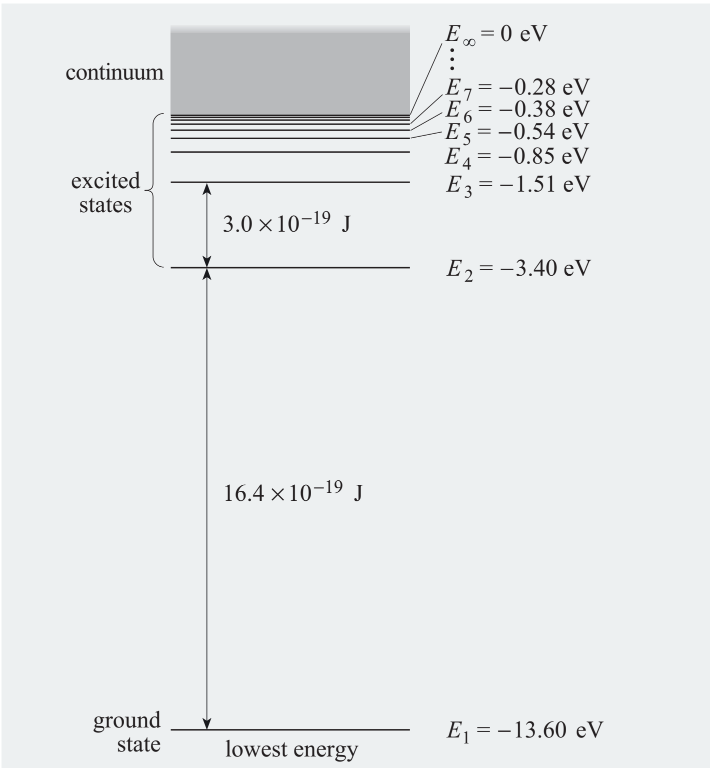


Figure 4.5 Energy levels of a hydrogen atom.

Notice that the energy levels in Figure 4.5 can be labelled either in SI units of joules or in electronvolts, eV. Notice, too, that the energies are all negative. This arises because it is convenient to define the energy to be zero when the electron is just detached from the nucleus. When it is attracted towards the nucleus it loses energy (like an apple falling to Earth loses gravitational potential energy) so all the energy levels within the atom must be negative.

An atom can only change its energy if it gains or loses exactly the right amount to make a transition to another energy level. The most common way to make a transition from an excited state to one of lower energy is for the atom to emit a photon. The photon energy E_{ph} is given by

$$E_{ph} = E_{high} - E_{low} \tag{4.7}$$

where E_{high} is the energy of the higher state and E_{low} that of the lower.

EXAMPLE 4.7

If a hydrogen atom makes a transition from level $n = 2$ to $n = 1$, what is the energy of the photon emitted?

The energy of the higher level is $E_{\text{high}} = -3.40\ \text{eV}$ and that of the lower level is $E_{\text{low}} = -13.6\ \text{eV}$.

From Equation 4.7

$$E_{\text{ph}} = -3.40\ \text{eV} - (-13.60\ \text{eV}) = 10.2\ \text{eV}$$

Since photon energy is related to the frequency and wavelength of electromagnetic radiation, each element has its own **emission spectrum**, in other words its own unique set of wavelengths that its excited atoms can emit. Most transitions between atomic energy levels involve radiation in the visible, infrared or ultraviolet region of the electromagnetic spectrum.

If an atom absorbs a photon whose energy exactly corresponds to the difference between its current energy level and a higher one, then **excitation** occurs, i.e. it makes a transition to the higher level. If a beam of radiation with a continuous range of wavelengths shines through a sample of atoms in low energy states, then photons will be absorbed if their energy corresponds exactly to the differences between energy levels. Radiation of wavelengths corresponding to these energies will therefore be removed from the beam, and an **absorption spectrum** will be observed. That is, there will be much less radiation at those wavelengths. These missing wavelengths are exactly the same as the wavelengths of the emission spectrum.

A more common way for atoms to become excited is by **collisional excitation**, i.e. in collisions with other atoms or with electrons. If the kinetic energy of the colliding particles is enough to cause excitation then some of their energy may be transferred to the atom and they will move apart with reduced kinetic energy.

If the atom receives sufficient energy, it is possible to dislodge an electron completely and produce an ion. The minimum energy needed to produce an ion, starting with the atom in its ground state, is called the ionization energy. On Figure 4.5, this corresponds to a transition from the level $n = 1$ to the top energy level $n = \infty$ or $E = 0$. If any more energy is transferred to the atom, then the transition corresponds to ending up in the region labelled 'continuum' on Figure 4.5, which means the electron and ion have some kinetic energy that enables them to move apart independently of one another.

EXAMPLE 4.8

In a certain region of the interstellar medium, the average kinetic energy of the particles is 12 eV. Are hydrogen atoms likely to become excited in collisions?

Yes they are. Excitation to the $n = 2$ level requires 10.2 eV (see Example 4.7) so collisions between particles with kinetic energy 12 eV are sufficiently energetic to cause excitation.

EXAMPLE 4.9

From Figure 4.5, what is the ionization energy of hydrogen?

To reach the energy level with $E = 0$, starting from the ground state, the atom must receive 13.60 eV or 21.79×10^{-19} J.

QUESTION 4.10

A hydrogen atom makes a transition from the energy level $n = 2$ to the level $n = 3$. State whether this will involve the emission or the absorption of a photon, and calculate its energy in eV.

QUESTION 4.11

At room temperature, the average particle kinetic energy is about 4×10^{-2} eV, or about 6×10^{-21} J (see Example 4.3). Explain whether hydrogen atoms at room temperature are likely to be in an excited state.

QUESTION 4.12

Explain what will happen if a beam of photons, all with energy 11.0 eV, shines onto a sample of hydrogen atoms (a) if all the atoms are initially in the $n = 2$ energy level (b) if all the atoms are initially in the ground state.

4.3.4 Subatomic particles

Atoms were at first thought to be **fundamental particles** — that is, the simplest building blocks from which all matter is constructed. It was thought that atoms were indivisible and did not contain any other particles. Then in the late 19th and early 20th centuries a different picture emerged: atoms are complex particles made up of protons, neutrons and electrons. The particles that make up atoms are **subatomic particles** — particles that are simpler than atoms.

Research during the 20th century revealed the existence of other subatomic particles, which are not found as constituents of atoms but are produced in radioactive decay, and/or detected in cosmic radiation (high-energy particles that enter the Earth's atmosphere and have extraterrestrial origin) and/or created in specially built particle accelerators, where mass–energy equivalence leads to the production of particles in high-energy collisions. Such particles would also be present in the very energetic conditions of the very early universe.

But are subatomic particles fundamental or are they in turn made up of simpler particles?

Electrons *are* believed to be fundamental. They belong to a family of six fundamental particles called **leptons**, listed in Table 4.2. The muon (whose symbol is the Greek letter mu, μ) and tauon (Greek tau, τ) are similar to the electron but have greater mass. The muon has about 200 times the mass of the electron and the tauon's mass is about 3500 times that of the electron. The superscript minus signs indicate that these particles have negative electric charge equal to the electron charge $-e = -1.60 \times 10^{-19}$ C.

Table 4.2 The leptons.

1st generation	2nd generation	3rd generation	Charge
electron, e^-	muon, μ^-	tau, τ^-	$-e$
electron-neutrino, ν_e	muon-neutrino, ν_μ	tau-neutrino, ν_τ	0

The three **neutrinos** (Greek nu, ν) associated with each of the electron, muon and tauon, are uncharged. The neutrinos have *extremely* small masses. It is currently (2002) believed that their masses are not quite zero but they have not yet been reliably determined. The three pairs (e.g. the electron and its neutrino) are often referred to as the three 'generations' of lepton.

To each lepton there is a corresponding **antiparticle**, an **antilepton**. An antiparticle is one with exactly the same mass as the corresponding particle but with charge (and some other properties) exactly opposite. The first antiparticle to be discovered was the antielectron, which is also called the **positron**. Table 4.3 lists the antileptons. The antileptons are denoted by symbols with superscript plus signs (indicating their charge) and the **antineutrinos** have a bar over the nu.

Table 4.3 The antileptons.

1st generation	2nd generation	3rd generation	Charge
positron, e^+	antimuon, μ^+	antitau, τ^+	$+e$
electron-antineutrino $\bar{\nu}_e$	muon-antineutrino, $\bar{\nu}_\mu$	tau-antineutrino, $\bar{\nu}_\tau$	0

The proton and neutron, however, are *not* fundamental particles. Each is made up of three **quarks**, which *are* believed to be fundamental. As with leptons, there are three 'generations' of quarks, which are shown in Table 4.4. The first generation of quarks have the lowest mass and the third are the most massive. Also like leptons, each quark has a corresponding **antiquark**, as listed in Table 4.5. Sometimes the symbols q and $\bar{q}$ are used to represent quarks and antiquarks in general without specifying a particular type.

Table 4.4 The quarks.

1st generation	2nd generation	3rd generation	Charge
up, u	charm, c	top, t	$+2e/3$
down, d	strange, s	bottom, b	$-e/3$

Table 4.5 The antiquarks.

1st generation	2nd generation	3rd generation	Charge
$\bar{u}$	$\bar{c}$	$\bar{t}$	$-2e/3$
$\bar{d}$	$\bar{s}$	$\bar{b}$	$+e/3$

Quarks have never been observed in isolation. They only seem to occur bound together, and particles made up of bound quarks are collectively known as **hadrons**. There are three sorts of hadron: three quarks can join to form a type of particle called a **baryon**, three antiquarks can form an **antibaryon**, and a quark and an antiquark can form a type of particle called a **meson**. All hadrons have charges that are either zero or whole-number multiples of the electron charge. All baryons have antiparticles, which are formed from the corresponding antiquarks.

EXAMPLE 4.10

A proton is composed of two ups and a down quark (uud), giving a total charge of $Q = 2e/3 + 2e/3 - e/3 = +e$, while a neutron has one up and two downs (udd), giving a net charge of zero.

The proton and neutron are both baryons, so the matter that makes up our Earth, and the other matter that we observe in the universe, is sometimes referred to as *baryonic matter*.

Apart for the proton and neutron, there are many other possible hadrons, though none is found as a constituent of ordinary matter. For example, a d quark and a $\overline{\mathrm{u}}$ antiquark form a meson called a π^-, which is found in cosmic radiation but is unstable so decays in a fraction of a second.

QUESTION 4.13

What is wrong with each of the following statements?

(i) Neutrons are fundamental particles.

(ii) Leptons combine to make hadrons.

(iii) All combinations of quarks and/or antiquarks are called baryons.

QUESTION 4.14

What are the constituents of an antiproton? What is the charge of an antiproton?

4.4 Chemical compounds

4.4.1 Symbols and formulae

A chemical element is a substance that is made up of just one type of atom. Each element has its own unique **chemical symbol** consisting of either one or two letters which are generally an abbreviation of its English or Latin name. The first letter of the symbol is always a capital, but the full name of the element is written in lower-case. For example, the symbol for aluminium is Al, while iron is Fe from the Latin *ferrum*. The symbol (e.g. Al or Fe) can stand either for a particular chemical element or for a single atom.

The **Periodic Table** (Table 7.2) lists all the known elements in order of increasing atomic number together with their symbols (and sometimes along with other properties). The layout of the table is such that elements listed in the same column have similar chemical properties.

A **chemical compound** is one in which two or more different types of atom are joined to create a different substance. The properties of a compound are generally quite unlike those of the elements from which it is made. For example, water is a compound of hydrogen and oxygen. The elements hydrogen and oxygen are both *gases* at room temperature while water is a liquid, and we need to breathe oxygen in order to stay alive but breathing water vapour (steam) is no substitute.

The **chemical formula** (or just the **formula**) of a substance is a shorthand way of describing its **chemical composition** — the different atoms it contains and the proportions in which they are present. Chemical symbols represent the atoms the substance contains and subscripts after the symbols indicate their numerical proportions.

EXAMPLE 4.11

The chemical formula for water is H_2O, showing that it is made up of hydrogen (H) and oxygen (O). The subscript 2 after the H means that there are two hydrogen atoms for each atom of oxygen.

EXAMPLE 4.12

Carbon monoxide has the symbol CO while carbon dioxide is CO_2. Both contain carbon (C) and oxygen (O), but the formula CO implies that there are equal numbers of carbon and oxygen atoms while in CO_2 the subscript 2 means that there are two oxygen atoms to each carbon atom.

EXAMPLE 4.13

The mineral haematite is a form of iron oxide and has the chemical formula Fe_2O_3. For every two atoms of iron (Fe) there are three of oxygen (O).

EXAMPLE 4.14

At room temperature, atoms of most gaseous elements tend to join together in pairs to make diatomic molecules. Oxygen gas has the formula O_2 to show that there are two atoms of oxygen joined together.

These examples show that the names of compounds can indicate their make-up (though that is not always the case). For example, compounds in which an element is combined with oxygen are collectively known as **oxides**, and the 'di' in dioxide indicates that there are two oxygens to each atom of the other element. But some oxides, such as water, are better known by other names.

As Example 4.12 indicates, the nature of the compound depends not only on the atoms it contains but also on the proportions in which they are combined — carbon dioxide is a normal part of the air we breathe, but even small quantities of carbon monoxide are toxic.

Example 4.13 also illustrates a convention when writing chemical formulae: if they contain a *metal* that symbol usually comes first.

QUESTION 4.15

The chemical formula for the compound calcium carbonate is $CaCO_3$. What are the numerical proportions of the atoms of calcium (Ca) carbon and oxygen that it contains?

QUESTION 4.16

Write down a chemical formula for the compound called silver perchlorate, which contains equal numbers of atoms of silver (Ag) and chlorine (Cl) and four atoms of oxygen for every atom of silver.

4.4.2 Chemical reactions

A **chemical reaction** is any process in which atoms become joined and/or separated to produce a different substance. Examples of chemical reactions include burning petrol in a car engine to produce exhaust gases, and the rusting or tarnishing of some metals when they are exposed to the mixture of substances that make up the air. Processes such as vaporization or dissolving are *not* chemical reactions because they do not produce different substances: when liquid water boils to become steam, it is still H_2O, and when sugar dissolves in water it is still present as sugar — it can be detected by its taste.

In any chemical reaction, atoms can only ever be rearranged and joined together in different ways; they are never created or destroyed. The atoms contained in the **products** of a reaction (what comes out) must be exactly the same as those contained in the **reactants**, as the starting substances are known.

A chemical reaction can be represented in a **chemical equation** (or reaction equation) using chemical symbols. Since the total number of atoms of each type is unchanged by a reaction, the total number of times each element's symbol appears (taking account of superscripts) must be the same on each side of a reaction equation, that is the equation must be balanced. For example, the most stable form of the element carbon is a black solid at room temperature (soot and charcoal are mainly carbon mixed with a few other substances). When it is heated with carbon dioxide, carbon monoxide is made. The equation for this reaction is

$$\mathrm{C} + \mathrm{CO_2} \longrightarrow 2\mathrm{CO} \tag{4.8}$$

The left-hand side of Equation 4.8 shows that when a single atom of carbon reacts with carbon dioxide to produce carbon monoxide, a total of two carbon atoms and two oxygen atoms are involved. Before the reaction, one carbon atom is present as pure carbon, while the other is combined with two oxygen atoms. After the reaction, the only substance present is the compound carbon monoxide (CO) and the 2 shows that there are two carbon and two oxygen atoms combined in this way so there are still two atoms of each type present, but they are arranged differently.

To produce a balanced equation from a description of a chemical reaction, it is best to start off by writing down the reactants and products with their correct formulae then adjust the numbers on each side to get a balance.

EXAMPLE 4.15

If the element copper (Cu) is heated in oxygen (O_2) there is a reaction that produces copper oxide (CuO) (which is a black powder). To produce a balanced equation, first write down

$$\mathrm{Cu} + \mathrm{O_2} \longrightarrow \mathrm{CuO}$$

There are two oxygen atoms on the left but only one on the right, so replace CuO by 2CuO:

$$\mathrm{Cu} + \mathrm{O_2} \longrightarrow 2\mathrm{CuO}$$

Now the oxygen atoms balance but there are two copper atoms on the right but only one on the left, so replace Cu by 2Cu to get a correctly balanced equation:

$$2\mathrm{Cu} + \mathrm{O_2} \longrightarrow 2\mathrm{CuO} \tag{4.9}$$

QUESTION 4.17

One of the main constituents of natural gas is the compound methane (CH_4). When methane burns it reacts with oxygen to produce carbon dioxide and water. Write a balanced equation for this reaction.

4.4.3 Ionic and molecular substances

Chemical compounds can be divided into two broad classes, according to the way in which their constituent particles are joined together.

Ionic substances are formed when metal elements, such as iron, sodium or copper react with non-metals such as chlorine, oxygen or nitrogen. Many minerals are ionic substances. Metal atoms are characterized by having low ionization energy, so they can easily lose one or more electrons to become positive ions. Non-metal atoms, on the other hand, can easily accommodate one or more extra electrons over and above the number required for electrical neutrality, so they readily form negative ions by taking electrons from metal atoms. The resulting positive and negative ions are then strongly attracted to one another by electrostatic forces (see 4.1.1). This way of joining particles together is called **ionic bonding**. The ions settle into a crystal structure where they are arranged in regular rows and layers. A good example of an ionic substance is sodium chloride, NaCl (table salt) shown in Figure 4.6. The compound contains equal numbers of positive sodium ions (Na^+) in a cubic pattern alternating with equal number of negative chloride ions (Cl^-). The arrangement of ions gives the crystal its shape: grains of salt are all tiny cubes.

As the ions are strongly attracted to one another, ionic substances are generally solid at normal atmospheric temperature and pressure: the ions need to acquire a large amount of internal energy before they can move from their positions in the crystal and so ionic substances are **non-volatile** — they have very high melting temperatures. At normal atmospheric pressure sodium chloride has a melting temperature of 801 °C and a boiling temperature of 1413 °C. When ionic substances are dissolved in water, or molten, the ions can move around freely and this enables them to conduct electricity.

When two non-metallic elements react, they generally form single **molecules** such as carbon dioxide (CO_2) or water (H_2O). A molecule is two or more atoms joined together in such as way that they share pairs of electrons. The outer electrons from the individual atoms go into orbits that encompass all the nuclei rather than being associated with just one. Many molecules consist of just a small number of atoms. Molecules with two atoms (**diatomic**) or three (**triatomic**) are common.

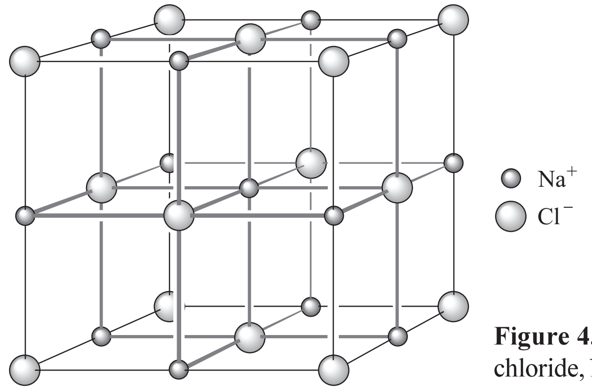


Figure 4.6 The structure of sodium chloride, NaCl.

Figure 4.7 Lewis structures of (a) chlorine atoms and (b) a chlorine molecule.

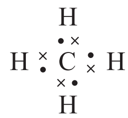


Figure 4.8 The Lewis structure of methane.

Joining particles by electron-sharing is called **covalent bonding** and is often represented schematically by a so-called **Lewis structure** after American chemist Gilbert Newton Lewis (1875–1946) in which dots and crosses represent the outer electrons (though in reality electrons are indistinguishable from one another). Figure 4.7 shows the Lewis structures for chlorine atoms and the covalently-bonded chlorine molecule Cl_2. The Lewis structure for methane (CH_4) is shown in Figure 4.8. Notice that in these examples the chlorine and carbon form molecules in which they are surrounded by eight outer electrons: this is a common pattern for covalent bonding involving atoms with atomic number (Z) less than 20, apart from hydrogen (Z = 1) which always shares just a single pair of electrons.

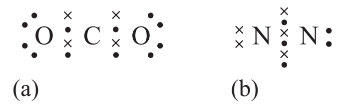


Figure 4.9 Lewis structures for (a) carbon dioxide, CO_2, and (b) nitrogen, N_2.

O=C=O (a)

N≡N (b)

Figure 4.10 Structural formulae for (a) CO_2 and (b) N_2.

Sometimes two or even three pairs of electrons may be involved in covalent bonding, as shown in Figure 4.9. When two pairs of electrons are shared, they form a **double bond** and the sharing of three pairs forms a **triple bond**. Single, double and triple covalent bonds are often represented by lines as shown in Figure 4.10, producing a representation known as a **structural formula**, which is simpler to draw than a Lewis structure but still shows which atoms are joined to one another — each line represents a shared pair of electrons.

QUESTION 4.18

Figure 4.11 shows the Lewis structures for water, H_2O, and ammonia, NH_3. Draw their structural formulae.

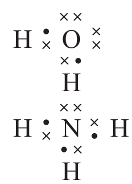


Figure 4.11 Lewis structures for water and ammonia.

QUESTION 4.19

Figure 4.12 shows the structural formula for tetrachloromethane (CCl_4). Draw its Lewis structure. (*Hint*: look at Figures 4.7 and 4.8 to deduce how many electrons are associated with a single atom of carbon and a single atom of chlorine.)

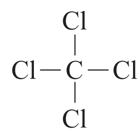


Figure 4.12 The structural formula for tetrachloromethane.

Covalent bonds within molecules are strong, but the forces that act between molecules are quite weak so **molecular substances** are generally **volatile** — they have low melting and boiling temperatures at normal atmospheric pressure. Water, with a melting temperature of 0 °C and a boiling temperature of 100 °C is an example, though many other molecular substances, such as carbon dioxide or oxygen (O_2) have even lower melting and boiling temperatures.

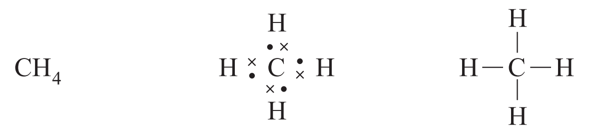
QUESTION 4.20

At normal atmospheric pressure, iron sulfide (FeS_2) has a melting temperature of 1171 °C, whereas hydrogen sulfide (H_2S) melts at −85°C. (i) What type of substance is FeS_2, and what sort of bonding occurs in solid FeS_2? (ii) What type of substance is H_2S, and what sort of bonding occurs in solid H_2S?

4.4.4 Organic molecules

Many carbon-based compounds form very large and complex molecules. Carbon-based compounds are collectively known as **organic molecules**, because many such compounds are produced naturally by organisms, i.e. by living things. Organic molecules are able to be large and complex because carbon atoms can form four covalent bonds with other atoms, including other carbon atoms. Carbon can thus form long chains while also bonding with other atoms.

The simplest type of organic molecules are those that are made up of just carbon and hydrogen. These are called **hydrocarbons**. The simplest hydrocarbon is methane, CH_4, whose molecular formula, Lewis structure and structural formula are shown in Figure 4.13. Hydrocarbons in which each carbon atom is bonded to its neighbour by a single bond are called **alkanes**.



molecular formula   Lewis structure   structural formula

Figure 4.13 The molecular formula, Lewis structure and structural formula of methane.

Alkanes in which the carbon atoms form chains are called **linear-chain alkanes**. In these molecules, each carbon is bonded to a maximum of two other carbon atoms and to two or (at the ends) three hydrogen atoms. Figure 4.14 shows the structural formula of the linear-chain alkane called butane (C_4H_{10}). Structural formulae of such molecules can become cumbersome, so we often use a simplified structural formula where only the bonds between carbon atoms are shown, so butane is represented as $CH_3—CH_2—CH_2—CH_3$. Butane is a relatively small hydrocarbon; linear-chain alkanes with over seventy carbon atoms are found in nature, and even longer ones can be made artificially.

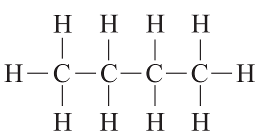


Figure 4.14 The structural formula of butane.

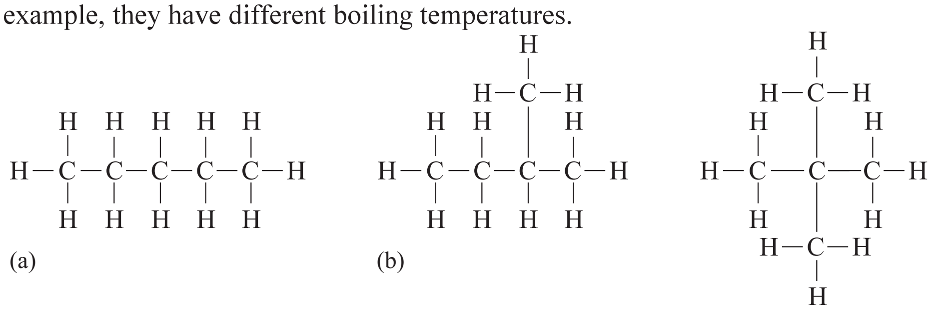
QUESTION 4.21

The molecule pentane, C_5H_{12}, contains a linear chain of five carbon atoms. Draw its full and abbreviated structural formulae.

QUESTION 4.22

Propane is an alkane that has a chain of three carbon atoms. Draw the full structural formula of propane and deduce its molecular formula.

Carbon's ability to form four covalent bonds means that a single molecular formula can correspond to more than one arrangement of carbon atoms. Different compounds that have the same molecular formula are called **isomers**. For example, Figure 4.15 shows the structural formulae of three isomers of C_5H_{12}. The isomers are different molecules each with their own structures and properties — for example, they have different boiling temperatures.



(a)   (b)   (c)

Figure 4.15 Three isomers of C_5H_{12}.

The compounds in Figure 4.15b and c are branched-chain alkanes, and a third type of alkane is shown in Figure 4.16 — this is a cycloalkane, in which the carbon atoms form a closed ring.

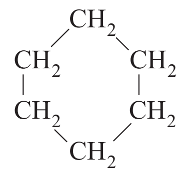


Figure 4.16 A cycloalkane, C_6H_{12}.

QUESTION 4.23

What are the molecular formulae of the structures shown in Figure 4.17?

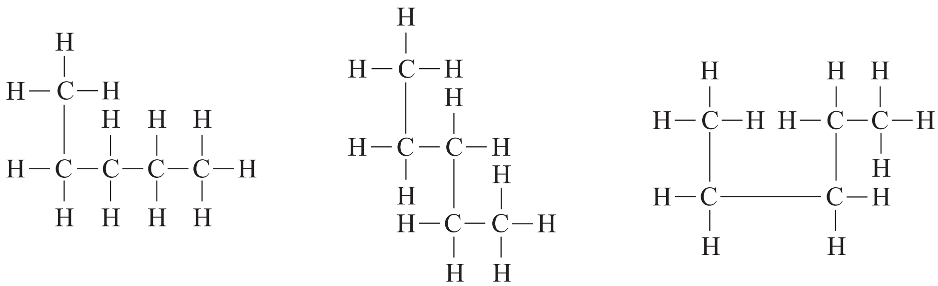
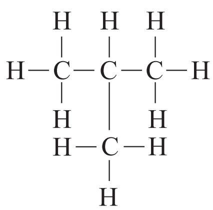


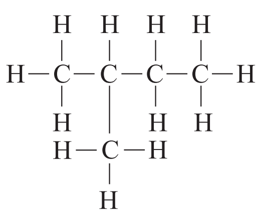
Figure 4.17 Diagram for Question 4.23.

Many organic compounds have both a traditional and a **systematic name**. The traditional name is generally quite short and may indicate a historical source of the compound. For example, citric acid was originally obtained from citrus fruits. The systematic name is given according to an internationally-agreed convention and conveys information about both its molecular and structural formulae. Figure 4.18 shows some hydrocarbons along with their systematic names. These particular compounds are all called 2-methyl something. The 'methyl' indicates the presence of $-CH_3$ (a so-called **methyl group**, from methane, see Figure 4.13) and the '2' tells us that the $-CH_3$ is attached to the second carbon atom in a chain. The final part of the name refers to the chain to which the methyl group is attached, such as pentane for a chain of five carbon atoms.

2-methylpropane



2-methylbutane



2-methylpentane

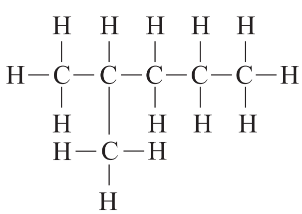


Figure 4.18 Structural formulae and systematic names of some branched-chain hydrocarbons.

Another important class of hydrocarbons contain a ring of six carbon atoms joined by alternating single and double bonds. These are called **aromatic compounds** (because many of them have a distinctive aroma, or smell). The simplest is benzene, C_6H_6, shown in Figure 4.19. Note that benzene is *not* an alkane because alkanes contain only single bonds. As with the alkanes, the systematic names of aromatic compounds can provide a convenient way to describe their structure. For example, the compound toluene, shown in Figure 4.20, has the systematic name methyl benzene, indicating that it has a methyl group attached to a benzene ring.

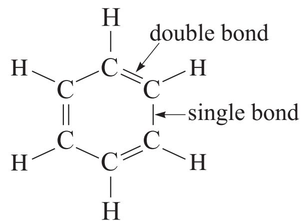


Figure 4.19 The structural formula of benzene.

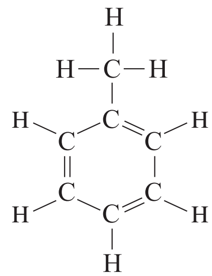


Figure 4.20 Toluene, systematic name methyl

QUESTION 4.24

Classify each of the structures in Figure 4.21 as a linear-chain alkane, a branched-chain alkane, a cycloalkane, an aromatic compound or a compound that is neither an alkane nor an aromatic compound.

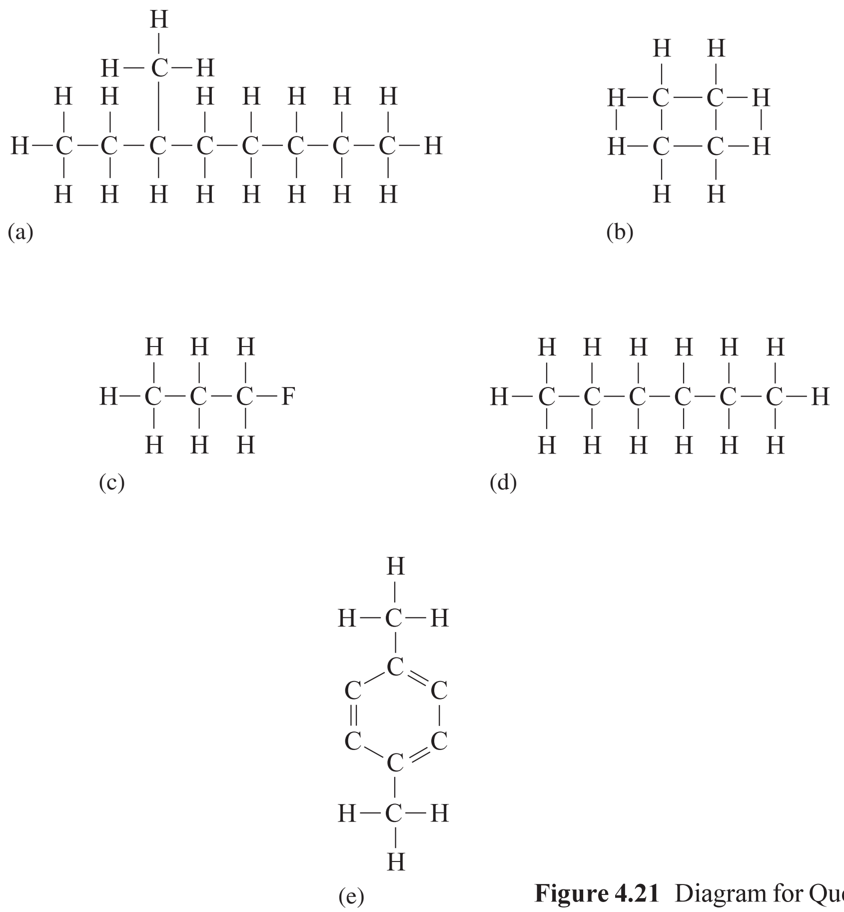


Figure 4.21 Diagram for Question 4.24.

4.5 Nuclear reactions

A **nuclear reaction** is any process in which the constituents of a nucleus combine or separate to produce a different nuclide. In all nuclear reactions, as in all other processes, three important **conservation laws** are always obeyed.

- Electric charge is conserved. The total charge of the products is exactly the same as that of the reactants.
- Mass number is unchanged. The total number of nucleons remains constant throughout the process.
- There is conservation of energy. In nuclear reactions, there are noticeable changes in mass and so mass–energy equivalence must be taken into account.

4.5.1 Fusion and fission

Two important types of nuclear reaction are *nuclear fusion* and *nuclear fission*. Nuclear fusion involves two light nuclei reacting to produce one with a greater mass number, whereas fission is the splitting of a nucleus with large mass number to form two lighter nuclei. In either case, the total mass of the products of the reaction is less than that of the reactants. The 'missing mass' is accounted for mass–energy equivalence and is manifest as an increase in overall kinetic energy of the particles involved and/or by the emission of high-energy photons (gamma-radiation).

Nuclear fusion reactions involve two nuclei getting close enough together to react. As nuclei all have positive electric charge, they repel one another by the electric force. Two nuclei can only approach closely enough to fuse if they have a lot of kinetic energy, which in turn requires a high temperature. It is therefore difficult to produce nuclear fusion reactions on Earth, but they do occur spontaneously in the very hot dense interiors of stars.

An important example of nuclear fusion is the series of reactions that provide the energy output from the Sun and many other stars. The net effect of these reactions is that hydrogen nuclei (protons) fuse to produce a nucleus of helium. The first step is the fusion of two protons to make a nucleus of deuterium along with a positron and a neutrino. The reaction can be represented using a nuclear equation:

$$ {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \longrightarrow {}^{2}_{1}\mathrm{H} + \mathrm{e}^{+} + \nu_{\mathrm{e}} \qquad (4.10)$$

where each of the hydrogen isotopes on the left is equivalent to a proton p.

Notice that the reaction equation is balanced in two important respects. First, the total of the mass numbers, indicated by the superscripts, is the same on both sides: on the left we have 1 + 1 and on the right we have 2. (The positron and neutrino have such small mass that they count as having mass number zero.) Second, the total electric charge is the same on both sides, as indicated by the atomic numbers (subscripts) of the nuclei and by the superscript + for the positron. On the left the total charge is 1 + 1 for the two protons and on the right it is still 1 for the deuterium plus 1 for the positron, and the neutrino is uncharged.

QUESTION 4.25

Equation 4.11 shows the next stage in the fusion of hydrogen to make helium. Verify that the equation is balanced. (Note that γ is a photon of gamma radiation.)

$$ {}^{2}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} \rightarrow {}^{3}_{2}\mathrm{He} + \gamma \tag{4.11} $$

QUESTION 4.26

The final stage of the production of helium involves two nuclei of ${}^{3}_{2}\mathrm{He}$ reacting to make one nucleus of ${}^{4}_{2}\mathrm{He}$ and some protons (which go on to take part in further reactions). By writing a balanced equation for the reaction, deduce how many protons must be produced.

Nuclear fission is less important than fusion in astronomy and planetary science, but is the process underlying the generation of nuclear power in power stations on Earth. Fission occurs in heavy nuclides, and is usually triggered by the absorption of a neutron to produce an isotope that breaks apart because it is unstable. The reactions exploited in nuclear power stations involve an isotope of uranium. One example produces a nucleus of barium (Ba) and one of krypton (Kr) and some neutrons.

$$ {}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{144}_{56}\mathrm{Ba} + {}^{90}_{36}\mathrm{Kr} + 2\,{}^{1}_{0}\mathrm{n} \tag{4.12} $$

Notice that the equation balances: the total mass number is 236 on each side and the total atomic number is 92.

QUESTION 4.27

The products of Equation 4.12 have more kinetic energy than the reactants. How can this be accounted for?

4.5.2 Radioactive changes

Certain nuclei are unstable, that is, they cannot survive forever in their present form but change (‘decay’) by emitting a particle and/or a photon. One nuclide can transform into another by this process of **radioactive decay**. Any substance containing unstable nuclei is termed **radioactive**, and the general name for the processes involved is **radioactivity**.

There are three types of radioactive decay, named alpha, beta and gamma (α, β and γ) after the first three letters of the Greek alphabet.

Alpha decay occurs when a *nucleus* with a fairly large mass number ejects an **alpha particle** (α-particle). An α-particle is the nucleus of a helium atom, a tightly-bound combination of two protons and two neutrons, and it is represented either as ${}^{4}_{2}\mathrm{He}$, or by the symbol α. A typical alpha decay is that of the isotope uranium-234 to produce thorium-230:

$$ {}^{234}_{92}\mathrm{U} \rightarrow {}^{230}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He} \tag{4.13a} $$

or, equivalently,

$$ {}^{234}_{92}\mathrm{U} \rightarrow {}^{230}_{90}\mathrm{Th} + \alpha \tag{4.13b} $$

Notice that the *mass numbers* add up to 234 on each side (mass number is conserved), and the atomic numbers add up to 92 on each side (*charge* is conserved). The actual total mass of the thorium and helium nuclei is slightly less than that of the uranium nucleus. This 'missing mass' is accounted for by energy conservation and *mass–energy equivalence*: the alpha particle is ejected at high speed so it has a lot of *kinetic energy*.

EXAMPLE 4.16

The thorium isotope produced in Equation 4.13 undergoes four further alpha decays to produce an isotope of lead (Pb). What are the atomic number and mass number of this isotope?

The total mass number of the four alpha particles is 16 and the total atomic number is 8. So the lead isotope must have mass number $A = 230 - 16 = 214$, and atomic number $Z = 90 - 8 = 82$. Its symbol is ${}^{214}_{82}\mathrm{Pb}$.

In **beta decay**, an *electron* or *positron* is produced within the nucleus and immediately ejected. The most common type of beta decay is beta-minus decay, where an electron is ejected. Here, a neutron in the nucleus converts into a proton along with an electron and an *antineutrino*:

$$ \mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^- + \overline{\nu}_\mathrm{e} \tag{4.14} $$

When this occurs within a nucleus, the electron and the antineutrino are immediately ejected with high energy. An electron that originates in this way is often called a **beta particle** (β-particle) or more accurately a beta-minus particle (β^--particle). The atomic number of the nucleus increases by 1 (it has one more proton than before) and its mass number remains unchanged. An example of a beta-minus decay is that of lead-214 to produce an isotope of bismuth:

$$ {}^{214}_{82}\mathrm{Pb} \rightarrow {}^{214}_{83}\mathrm{Bi} + \mathrm{e}^- + \overline{\nu}_\mathrm{e} \tag{4.15} $$

Notice that the mass number is 214 on each side (the electron and neutrino each have a mass number zero), and the atomic number is 82 on the left, and on the right $Z = 82$ (Bi) $-$ (-1)(electron) $= 83$ as required.

Rather less common is beta-plus decay, when a *positron* and an electron neutrino are ejected and the atomic number *decreases* by 1. The unstable isotope oxygen-14 decays in this way:

$$ {}^{14}_{8}\mathrm{O} \rightarrow {}^{14}_{7}\mathrm{N} + \mathrm{e}^+ + \nu_\mathrm{e} \tag{4.16} $$

EXAMPLE 4.17

The caesium isotope ${}^{137}_{55}\mathrm{Cs}$ decays to produce the barium isotope ${}^{137}_{56}\mathrm{Ba}$. What type of decay is involved, and what is the complete reaction equation?

This is beta-minus decay, since the atomic number increases by 1. The equation is

$$ {}^{137}_{55}\mathrm{Cs} \rightarrow {}^{137}_{56}\mathrm{Ba} + \mathrm{e}^- + \overline{\nu}_\mathrm{e} \tag{4.17} $$

The final type of decay is gamma decay. Unlike alpha decay and beta decay this involves no changes in the numbers of nucleons. Gamma decay occurs when a nucleus finds itself in an *excited state*, meaning that the nucleons are arranged in such a way as to have excess energy. A nucleus can lose this energy by emitting a *photon* of gamma-radiation and so reach its *ground state* (its minimum energy). The energy of the photon is typically several hundred keV (eV are *electronvolts*).

A nucleus in an excited state is sometimes indicated by an asterisk (*). A gamma decay often occurs following alpha decay or beta decay. For example, the decay shown in Equation 4.17 leaves the barium in an excited state so it undergoes gamma decay:

$$^{137}_{56}\mathrm{Ba}^* \rightarrow {}^{137}_{56}\mathrm{Ba} + \gamma \tag{4.18}$$

QUESTION 4.28

The thorium isotope $^{234}_{90}\mathrm{Th}$ decays to produce the uranium isotope $^{234}_{92}\mathrm{U}$. What change(s) must have taken place in the nucleus and what particle(s) must be emitted?

QUESTION 4.29

The radium isotope $^{226}_{88}\mathrm{Ra}$ decays to produce the radon isotope $^{222}_{86}\mathrm{Rn}$. What change(s) must have taken place in the nucleus and what particle(s) must be emitted?

4.5.3 Radioactive decay

A sample of radioactive material will gradually change its chemical composition as nuclei change from one to another by the process of alpha and beta decay. For example, in a sample that is initially pure uranium-234, the number of ^{234}U nuclei will decrease over time as they undergo alpha decay to be replaced by thorium-230 as shown in Equation 4.13.

Radioactive decay is an intrinsically random process and we can never know when an individual nucleus will decay. However, if we have a large enough sample of nuclei we can determine the time over which *half* of them will decay. This is known as the radioactive **half-life** of the sample and it is related to the probability of an individual nucleus decaying.

The half-life of a nuclide is a property of the nucleus itself and is not affected by any external factor: half-life is unaffected by conditions such as temperature or pressure, nor is it influenced by any chemical reactions. A sample of uranium-234 nuclei will have *exactly* the same half-life regardless of whether they are inside uranium atoms that are moving around independently in a *gas*, or combined with other atoms in *minerals* buried deep underground.

In the laboratory, half-lives of radioactive materials can be measured by monitoring their **activity**. The activity of a radioactive sample is the number of decays it undergoes per second, i.e. the number of alpha or beta particles that it emits per second. The activity is *directly proportional* to the number of unstable nuclei still present in the sample, thus if the number of unstable nuclei halves, then so does the activity. A graph of activity against time can therefore be used to deduce half-life. Figure 4.22 shows a schematic graph of activity against time for a sample of the isotope radon-220. Notice that the activity halves in each time interval of 52 seconds (the half-life).

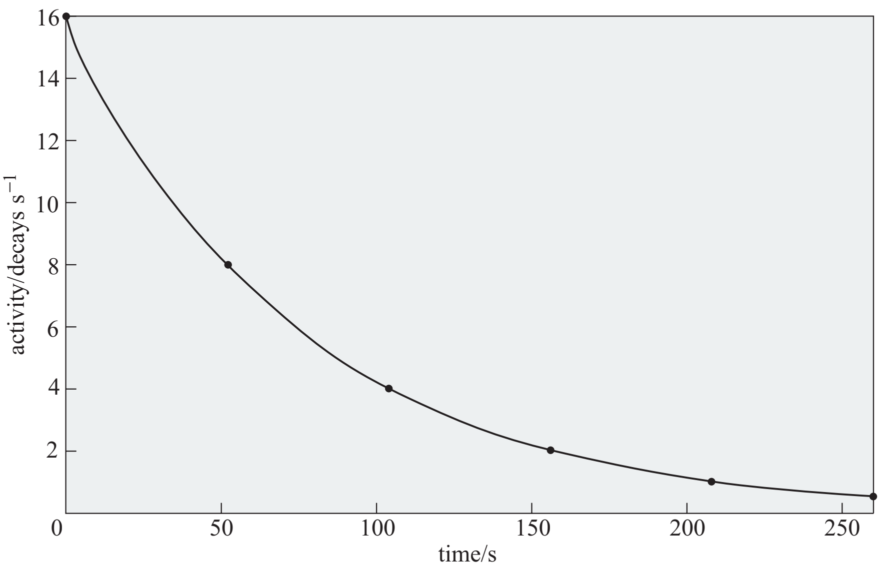


Figure 4.22 The activity of a sample of radon-220 decreases with time.

Radioactive half-lives can range from small fractions of a second (for very unstable nuclides) to many millions of years for some long-lived isotopes. The fact that half-life is unaffected by external conditions means that the number of unstable nuclei remaining in a sample can be directly related to its age. Provided the nuclei in question are not replenished (e.g. by the decay of some other nuclide) then they can be used to date a sample as the following examples illustrate.

Note that in both S282 and S283 we frequently use 1 Ma for a time span of one million years (10^6 years) and 1 Ga for one thousand million years (10^9 years).

EXAMPLE 4.18

The isotope uranium-238 has a half-life of 4.5 Ga, which is close to the age of the Earth. Of the uranium-238 that was present in rocks when the Earth formed, how much is left? How much will be left when the Earth is twice its present age?

As a period equal to the half-life has elapsed, the number of uranium-238 nuclei will have halved. In a further 4.5 Ga, the number will halve again, so the amount of uranium-238 remaining will be one-quarter of that which was present when the Earth formed.

EXAMPLE 4.19

The isotope potassium-40 is found in rocks. It decays with a half-life of 2.4×10^8 years to produce argon gas which remains trapped in the rock. In a certain rock sample, it is found that there are 15 argon atoms for every potassium atom. Assuming that the rock originally contained no argon, how old is the rock?

Originally all the argon atoms must have been potassium, so for every sixteen potassium atoms, fifteen have now become potassium and the number of potassium atoms has dropped to one-sixteenth the original number. So the number has halved, halved again (to one-quarter), halved again (one-eighth) and halved again (one-sixteenth). Four half-lives have elapsed, so the age of the rock is $4 \times 2.4 \times 10^8$ years = 9.6×10^8 years.

QUESTION 4.30

Radium-226 has a half-life of 1600 years. What fraction of a given sample will remain after 6400 years?

QUESTION 4.31

A certain isotope of silver has a half-life of 20 minutes. How long does it take for the activity of a sample to decrease to one-eighth of its initial activity?

4.6 Answers and comments for Topic 4

QUESTION 4.1

(i) and (ii) Both aluminium and nitrogen can exist in any of the three states, depending on the temperature and pressure.

(iii) A gas is a plasma only if most of its atoms are ionized and these ions and electrons can move freely.

QUESTION 4.2

From Equation 4.1,

$$\begin{aligned}\rho &= \frac{m}{V}\\ &= \frac{2.34\,\text{kg}}{5.22\times10^{-4}\ \text{m}^{-3}}\\ &= 4.48\times10^{3}\ \text{kg m}^{-3}\end{aligned}$$

QUESTION 4.3

Rearranging Equation 4.2,

$$\begin{aligned}n &= \frac{\rho}{M}\\ &= \frac{1\times10^{5}\ \text{kg m}^{-3}}{1.67\times10^{-27}\ \text{kg}}\\ &= 6\times10^{31}\ \text{m}^{-3}\end{aligned}$$

QUESTION 4.4

From Equation 4.3, T = 727°C.

Using Equation 4.4 and remembering to have T in K,

$$\begin{aligned}E_k &= \frac{3kT}{2}\\ &= \frac{3\times1.38\times10^{-23}\ \text{J K}^{-1}\times1000\ \text{K}}{2}\\ &= 2\times10^{-20}\ \text{J}\end{aligned}$$

(an approximate answer, because we are only given an approximate temperature).

QUESTION 4.5

The average kinetic energy will be the same for each different type of atom, because average kinetic energy depends only on the temperature. But the hydrogen atoms will have the highest average speeds because they have the lowest mass.

QUESTION 4.6

From Equation 4.6, $p = nkT$

$$= 9.0 \times 10^{26}\,\mathrm{m^{-3}} \times 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}} \times 733\,\mathrm{K}$$

$= 9.1 \times 10^{6}$ Pa (i.e. about ninety times the pressure at the Earth's surface).

QUESTION 4.7

(i) Losing two electrons leads to a net positive charge twice that of a proton so the symbol is Fe^{2+}. (ii) The ion Cl^{-} has a negative charge equal to that of a single electron, so the atom must gain an electron.

QUESTION 4.8

The number of protons is $Z = 26$. The number of nucleons (protons plus neutrons) is $A = 56$, so there must be 30 neutrons.

QUESTION 4.9

There are 56 protons and mass number A = (no. of protons) + (no. of neutrons) = 81 + 56 = 137 so the isotope is ${}^{137}_{56}\mathrm{Ba}$.

QUESTION 4.10

The atom's energy increases so it absorbs a photon. Using Equation 4.7, the photon's energy is $E_{\mathrm{ph}} = -1.51\,\mathrm{eV} - (-3.40\,\mathrm{eV}) = 1.89\,\mathrm{eV}$.

QUESTION 4.11

The average kinetic energy of collisions is far too small to excite the hydrogen atoms to the $n = 2$ level, since this requires just over 10 eV (Example 4.7 and Figure 4.5), so they will be in the ground state.

QUESTION 4.12

(a) The photon energy is more than the 3.40 eV needed to make the transition from $n = 2$ to $n = \infty$ so the photons will be able to ionize the atoms. The photons will therefore be absorbed, producing hydrogen ions and electrons with enough kinetic energy to move independently of one another.

(b) The beam will pass straight through. This is because the least energetic photon that can be absorbed by hydrogen in the ground state corresponds to a transition to the $n = 2$ level, and this requires exactly 10.2 eV (see Example 4.7). The next transition is from the ground state to the $n = 3$ level, and this requires energy of exactly $-1.51\,\mathrm{eV} - (-13.60\,\mathrm{eV}) = 12.09\,\mathrm{eV}$. The photon energy of 11.0 eV matches neither of these so the photons cannot be absorbed.

QUESTION 4.13

(i) Neutrons are not fundamental particles; they are made up of quarks.

(ii) Hadrons are made up of quarks, not leptons.

(iii) Only three-quark combinations are called baryons.

QUESTION 4.14

An antiproton is made up of $\bar{u}\,\bar{u}\,\bar{d}$, and its charge is $Q = -2e/3 - 2e/3 + e/3 = -e$.

QUESTION 4.15

There are equal numbers of calcium and carbon atoms, and for every atom of calcium there are three of oxygen.

QUESTION 4.16

$AgClO_4$

QUESTION 4.17

The compounds are $CH_4 + O_2 \longrightarrow CO_2 + H_2O$.

First balance the hydrogen:

$$CH_4 + O_2 \longrightarrow CO_2 + 2H_2O$$

then the oxygen:

$$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$$

QUESTION 4.18

See Figure 4.23.

Figure 4.23 Structural formulae for water and ammonia. The answers to Question 4.18.

QUESTION 4.19

See Figure 4.24.

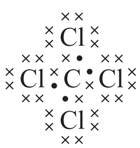


Figure 4.24 The Lewis structure for tetrachloromethane. The answer to Question 4.19.

QUESTION 4.20

(i) FeS_2 is a compound of a metal with a non-metal, and is non-volatile so we can deduce that it is an ionic substance with a crystal structure, held together by ionic bonding.

(ii) H_2S is a volatile compound of two non-metal elements so we can deduce that it is a molecular substance. The atoms within the molecules are joined by covalent bonds, but the forces acting between molecules in the solid state are weak.

QUESTION 4.21

See Figure 4.25. The abbreviated structural formula of pentane is $CH_3{-}CH_2{-}CH_2{-}CH_2{-}CH_3$.

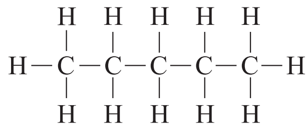


Figure 4.25 The structural formula of pentane. The answer to Question 4.21.

QUESTION 4.22

See Figure 4.26. The molecular formula of propane is C_3H_8.

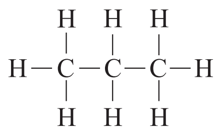


Figure 4.26 The structural formula of propane. The answer to Question 4.22.

QUESTION 4.23

All the structures have the molecular formula C_5H_{12}.

QUESTION 4.24

(a) Branched-chain alkane, (b) cycloalkane, (c) neither an alkane nor aromatic (it contains fluorine, F, so is not an alkane), (d) linear-chain alkane, (e) aromatic compound.

QUESTION 4.25

Total mass numbers: 1 + 2 (left side) = 3 (right)

Total charge: 1 + 1 (left) = 2 (right)

The photon, γ, has neither charge nor mass. The equation is balanced.

QUESTION 4.26

There must be two protons, because the mass numbers must add up to 6 each side and the charges (atomic numbers) must add up to 4:

$${}^{3}_{2}\mathrm{He} + {}^{3}_{2}\mathrm{He} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H}$$

QUESTION 4.27

The total mass of the products must be less than that of the reactants. *Mass–energy equivalence* ensures that the increase in energy is equivalent to the loss of mass.

QUESTION 4.28

There has been no change in mass number so no alpha decay is involved. The atomic number has increased by 2, indicating that two neutrons have converted into protons in the nucleus and two beta-minus particles (electrons) have been ejected.

QUESTION 4.29

The mass number must decrease by 4 and the atomic number by 2 so this indicates α-decay, and the reaction equation is:

$$^{226}_{88}\mathrm{Ra} \rightarrow \, ^{222}_{86}\mathrm{Rn} + \, ^{4}_{2}\mathrm{He}$$

QUESTION 4.30

6400 years is four times the half-life, so the number of radium-226 nuclei will have halved four times and one-sixteenth of the original number will remain.

QUESTION 4.31

Three half-lives must have elapsed, because the activity halves, halves again (to one-quarter of its initial value) and halves again (to one-eighth). A time of one hour must have elapsed.

TOPIC 5
MINERALS AND ROCKS

5.1 Rock formation and features

Note to S282 students: this section relates to material in S283.

A **rock** is a *solid* assembly of mineral grains. A **mineral** is a solid material, formed by a natural process, with a *chemical composition* that falls within certain narrow limits. Its constituent atoms are arranged in a regular three-dimensional array and this determines the characteristic shape of the *crystal*. A rock may consist of just one type of mineral, but more usually it contains several different types. Mineral grains can be intact crystals or fragments, and can vary in size from a few micrometres to a few centimetres.

Different types of rocks form in different ways, and the processes of formation and any subsequent activity leave their marks on the rocks. The rocks that make up the Earth's surface are manifestations of the Earth's activity, and on other rocky bodies (planets or natural satellites) so rocks provide important clues to any activity or lack thereof. There are three main processes of rock formation, each of which produces characteristic features in the resulting rock.

Rocks that have solidified from a molten state are **igneous rocks**. On Earth, heating deep in the planet's interior to temperatures around 1000 °C produces **magma** (which is molten rock). Magma may emerge onto the Earth's surface via a volcanic eruption, in which case it is known as **lava**. The rock thus formed is called an extrusive igneous rock (Figure 5.1a). Alternatively magma may cool slowly while still underground forming so-called **intrusive** igneous rock as in Figure 5.1b. As the magma cools, crystals grow from the liquid. The rocks that form underground may eventually be uncovered, as overlying rocks become worn away and deposited elsewhere. Igneous rocks are characterized by the presence of complete crystals and their size indicates the rate at which the magma cooled: in general, large crystals are produced by slow cooling.

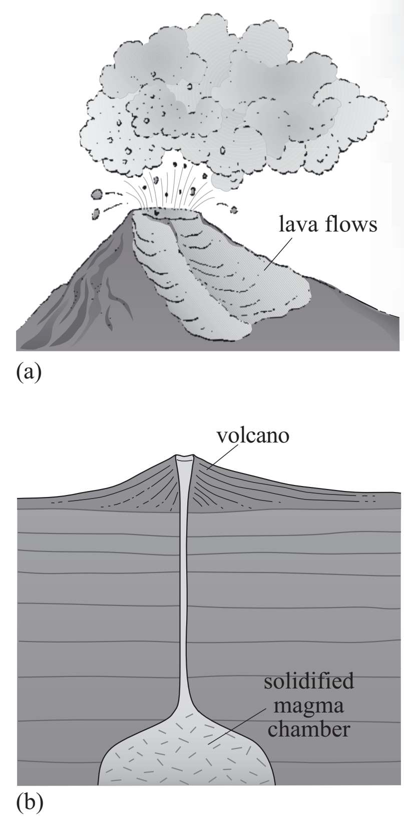


Figure 5.1 Igneous rock forms when (a) magma erupts onto the Earth's surface or (b) cools underground.

Sedimentary rocks are formed by the deposition of layers of sediments. The sediment might originate if rocks have been broken up by **weathering** (exposure to rain, wind and frost) and the resulting small fragments have been transported by water, wind or glaciers to be deposited elsewhere in roughly horizontal layers known as **strata**. The strata often include shells and skeletons of marine organisms. The grains of sedimentary rocks are usually fragments rather than complete crystals, and the presence of fossils generally indicates a sedimentary rock.

The third main group of rocks is called **metamorphic** (meaning 'changed form'). A metamorphic rock is formed when any type of rock is heated to temperatures of several hundred degrees Celsius and/or subjected to high *pressure* because of the *weight* of overlying rocks. Unlike igneous rocks, metamorphic rocks do not cool from a *liquid*; rather, the change occurs while the rock remains in the *solid* state.

During metamorphism, the atoms in the minerals making up the rock become reorganized, sometimes resulting in the formation of new minerals and changing the rock's appearance while maintaining the same *chemical composition*. Metamorphic rocks, like igneous rocks, are crystalline, but the process of metamorphism often results in banding or alignment of minerals.

QUESTION 5.1

What is wrong with each of the following statements?

(i) A mineral is another name for a rock.

(ii) A rock that contains whole crystals is likely to be a sedimentary rock.

(iii) Igneous and metamorphic rocks differ only in the way they were formed and are indistinguishable in appearance.

5.2 Common minerals and rocks

5.2.1 Rock-forming minerals

Table 5.1 The composition of the Earth's crust.

Element	Symbol	% by mass
oxygen	O	46.6
silicon	Si	27.7
aluminium	Al	8.3
iron	Fe	5.0
calcium	Ca	3.6
sodium	Na	2.8
magnesium	Mg	2.1
all others		1.3

More than 3500 different *minerals* have been identified at the Earth's surface, but the number of common *rock*-forming minerals is much smaller. The *chemical composition* of a mineral depends on the *elements* available for its formation. Table 5.1 lists the most abundant elements in the Earth's crust.

Minerals containing oxygen combined with silicon are the most common minerals at the Earth's surface. Some of these, such as quartz, are almost entirely silicon dioxide (SiO_2) with traces of other elements, but the most common are **silicates**, in which silicon and oxygen are grouped together as SiO_4 and combined with one or more metals.

Other important minerals include **carbonates** (containing the CO_3 group). The mineral calcite is calcium carbonate. On Earth the *sedimentary* rocks limestone and chalk, which can originate from the shells of living organisms (chalk and some limestones) or by chemical precipitation (some limestones), are also largely calcium carbonate. Many minerals are *oxides* (such as haematite) which contain metal atoms combined with oxygen.

While minerals have a fairly well-defined chemical make-up, many crystal structures can accept small variations in composition. For example, the mineral olivine is composed of magnesium and/or iron combined with the silicate group SiO_4. Its *chemical formula* is written $(Mg, Fe)_2SiO_4$, showing that for every SiO_4 group there are two metal atoms, but the proportions of magnesium and iron can vary from specimen to specimen.

Mineral formation is also affected by physical conditions (temperature and pressure) which influence the *crystal* structure adopted by the atoms. It is therefore possible to have two or more different minerals with the same chemical composition, but with different structures.

QUESTION 5.2

Suggest at least two reasons why minerals are generally referred to by name (e.g. quartz, haematite) rather than by their chemical formulae.

5.2.2 Common rock types

On Earth, the most common *igneous* rock types include **basalt** and **granite**. Basalt is an *extrusive* igneous rock, formed by rapid cooling and crystallization. Its rapid cooling leads to a fine grain structure. Chemically it contains quite a high proportion of the metals magnesium, calcium and iron, combined in various minerals with the elements oxygen and silicon.

Granite, by contrast, is in an *intrusive* igneous rock which has cooled slowly underground. It contains relatively small amounts of calcium, iron or magnesium, but quite large proportions of silicon, sodium and potassium. Its slow cooling ensures that the mineral grains are large, giving the rock a coarse texture.

Sedimentary rocks rely on a variety of processes for their formation. On Earth, rain, wind and glaciers bring about weathering and erosion, and then the transportation of the resulting fine fragments which are deposited elsewhere to form new rocks. Common sedimentary rocks on Earth include (in order of decreasing grain size) **conglomerates**, **sandstones** and **mudstones**. Chemically, their composition varies according to that of the original rock. Conglomerates contain mainly pebbles and rock fragments. Sands are composed mainly of mineral grains (especially quartz) or broken shells of organisms. The shelly material consists largely of calcium carbonate and forms **limestone** deposits. Muds contain clay minerals (which are *silicates*) and organic matter and, if buried under further deposits, become compacted to form **shale**.

Slate and **marble** are examples of *metamorphic* rocks found on Earth. Slate originates from shale, a sedimentary rock that has been heated to 200 to 350 °C at depths of 5 to 10 km below the Earth's surface. Heating at greater temperatures and pressures produces **gneiss**, which has much coarser grains than slate but whose chemical composition still reflects that of the silicates of the original mud. Marble originates from limestone and generally contains only one mineral, calcite, which is a form of calcium carbonate.

QUESTION 5.3

Complete Table 5.2 by putting a tick (✓) in appropriate cells so that the table summarizes the origins and some key features of the rock types listed.

Table 5.2 Table for Question 5.3.

	limestone	granite	slate	basalt	marble
igneous					
sedimentary					
metamorphic					
fine-grained texture					
contains mainly carbonates					

5.3 Answers and comments for Topic 5

QUESTION 5.1

(i) A rock is made up of minerals, usually a mixture of several types. A mineral has a well-defined chemical composition and a crystal structure, but a rock has neither.

(ii) A rock containing whole crystals is like to be igneous or metamorphic. A sedimentary rock is more likely to be made up of crystal fragments.

(iii) Igneous and metamorphic rocks do indeed form in different ways: igneous from molten rock and metamorphic from changes brought about in the solid state. Metamorphic rocks can usually be distinguished from igneous rocks by the banding or alignment of crystals found in metamorphic rocks.

QUESTION 5.2

Reasons include:

- the chemical composition of some minerals can vary slightly from specimen to specimen;
- minerals may have quite different properties arising from their crystal structure while having the same chemical composition;
- mineral names tend to be shorter and more memorable than complex chemical formulae.

QUESTION 5.3

See Table 5.3.

Table 5.3 The answer to Question 5.3.

	limestone	granite	slate	basalt	marble
igneous		✓		✓	
sedimentary	✓				
metamorphic			✓		✓
fine-grained texture	*		✓	✓	*
contains mainly carbonates	✓				✓

* Many limestones and marbles can have a fine-grained texture.

TOPIC 6
MATHEMATICS

6.1 Powers and logarithms

6.1.1 Powers

We use the notation of 'powers' as a shorthand in both *arithmetic* and *algebra*. For example, 10^3 means $10 \times 10 \times 10$ and r^2 means $r \times r$. The superscript (3 or 2 in these examples) is the 'power' to which the number or quantity is raised, and is called the **index** or **exponent**.

When there is a multiplication involving the same number (or quantity) raised to two different powers, the exponents (indices) are added together. For example,

$$10^3 \times 10^2 = (10 \times 10 \times 10) \times (10 \times 10)$$
$$= 10^5 = 10^{(3+2)} \qquad (6.1)$$

and

$$4r^2 \times 3r^3 = 3 \times 4 \times r^2 \times r^3 = 12r^5 \qquad (6.2)$$

Similarly, when there is a division the exponents are subtracted from one another:

$$10^5/10^2 = 100\,000/100 = 10^3 = 10^{(5-2)} \qquad (6.3)$$

and

$$20r^3/4r^2 = 5r = 5r^{(3-2)} \qquad (6.4)$$

As Equation 6.4 shows, raising any quantity to the power of 'one' leaves it unchanged.

If the two powers are the same, the result gives a meaning to a power of zero. For example:

$$r^3/r^3 = r^{(3-3)} = r^0 \qquad (6.5a)$$

but we also can see that

$$r^3/r^3 = 1 \qquad (6.5b)$$

Comparing both versions of Equation 6.5 we can see that $r^0 = 1$. Raising *any* quantity or number to the power of zero gives a result of one.

When a quantity is raised to one power and then another, the powers are multiplied together. For example:

$$(r^3)^2 = (r \times r \times r) \times (r \times r \times r) = r^6 = r^{(3 \times 2)} \qquad (6.6)$$

QUESTION 6.1

Write the following expressions in as compact a form as possible.

(i) $3a^3 \times 5a^4$ (ii) $6b^5 \div 2b^2$ (iii) $3c \times 4c^2 \div 2c^3$ (iv) $(2d^2)^4$

6.1.2 Negative exponents

The pattern of division can be extended to give a meaning to **negative exponents**, for example:

$$10^3/10^5 = 10^{(3-5)} = 10^{-2} \tag{6.6a}$$

and similarly

$$10^3/10^5 = 1000/100\,000 = 1/100 = 1/10^2 \tag{6.6b}$$

Comparing both versions of Equation 6.6 we can see that raising a quantity to negative exponent must be interpreted as 'one over' the same quantity raised to the positive exponent, i.e. its **reciprocal**.

To find the reciprocal of a fraction, just turn it 'upside down'. For example,

$$\frac{1}{2/5} = \frac{1}{0.4} = 2.5 = \frac{5}{2}$$

For any quantities p and r

$$\frac{1}{p/r} = \frac{r}{p} \tag{6.7}$$

Finding the reciprocal of a reciprocal gets you back to where you started:

$$\frac{1}{5} = 0.2 \text{ and } \frac{1}{0.2} = 5$$

For any quantity r,

$$1/(1/r) = r \tag{6.8a}$$

or, using exponents

$$(r^{-1})^{-1} = r^1 = r \tag{6.8b}$$

For any quantity expressed purely as a power, the reciprocal is found by changing the sign of the exponent:

$$1/r^a = (r^a)^{-1} = r^{-a} \tag{6.9}$$

If there is another number multiplying the 'power' then you need to find its reciprocal as well. For example

$$1/(5 \times 10^3) = (5 \times 10^3)^{-1} = 5^{-1} \times 10^{-3} = 0.2 \times 10^{-3}$$

(which could also be written as $10^{-3}/5$ or as 2×10^{-4}).

Reciprocals can easily be found on a calculator using the $1/x$ button. Enter the number whose reciprocal you want to find (i.e. x) then press $1/x$.

QUESTION 6.2

Write each of the following expressions in as compact a form as possible.

(i) $1/(0.5a^4)$ (ii) $(3b^2)^{-1}$

QUESTION 6.3

Use the $1/x$ button on a calculator to evaluate each of the following reciprocals.

(i) $1/125$ (ii) 35^{-1}

6.1.3 Fractional exponents

We can give a meaning to **fractional exponents**. For example, a square with sides of length d has area A where

$$d^2 = A$$

taking the square root of each side we can write

$$d = \sqrt{A}$$

Taking the **square root** means halving the exponent of d^2 to get just d (i.e. d^1). Since we must always do the same thing to both sides of an *equation* we must also halve the exponent of A:

$$d = A^{1/2} = A^{0.5} = \sqrt{A} \qquad (6.10)$$

(Notice that the power can be written either as a fraction or a decimal.)
Some other fractional exponents also have an obvious meaning. For example, $x^{-1/2}$ is the reciprocal of $\sqrt{x}$, and $x^{1.5}$ is x^2 divided by $\sqrt{x}$, as

$$x^2/x^{1/2} = x^2 \times x^{-0.5} = x^{1.5} \qquad (6.11a)$$

or, which comes to exactly the same thing, it is the square root of x^3

$$(x^3)^{1/2} = x^{3/2} = x^{1.5} \qquad (6.11b)$$

Fractional exponents can be combined in multiplication and division in exactly the same way as those that are whole numbers.

Square roots can be found on a calculator using the $\sqrt{x}$ button. Key in the number then press $\sqrt{x}$. A number raised to any exponent can be found using the y^x button. For example, to calculate $7^{0.37}$, enter 7, then press y^x then enter 0.37 to get the answer 2.054….

Note that some functions on your calculator may need you to press the SHIFT key first. For example on many calculators y^x is often operated by pressing SHIFT then y^x.

QUESTION 6.4

Write the following expressions in as compact a form as possible.

(i) $2a^{1.5} \times 3\sqrt{a}$ (ii) $6b/2\sqrt{b}$ (iii) $\sqrt{4c^2/d^2}$

QUESTION 6.5

Use the or y^x button on a calculator, as appropriate, to evaluate each of the following expressions.
(i) $10^{0.5}$ (ii) 17^3 (iii) $2.4^{1.7}$

6.1.4 Scientific notation

In astronomy and planetary science we sometimes need to deal with very large or very small numbers. Rather than writing out strings of zeroes we usually express such numbers in **scientific notation** (which is also known as ‘standard form’). In this notation, all numbers are expressed as some number with just one figure before the decimal point, multiplied by a power of ten.

EXAMPLE 6.1

There are said to be 1460 000 000 000 000 000 000 litres of water stored on Earth. How can this be expressed in scientific notation?

The quantity of water can be written as
$1.46 \times 1000\,000\,000\,000\,000\,000\,000$ litres $= 1.46 \times 10^{21}$ litres.

EXAMPLE 6.2

The mass of a proton is 1.67×10^{-27} kg. What is this when written out in full?

$10^{-27} = 0.000\,000\,000\,000\,000\,000\,000\,000\,001$, so the mass of a proton is $0.000\,000\,000\,000\,000\,000\,000\,000\,001\,67$ kg.

As Examples 6.1 and 6.2 illustrate, scientific notation provides a compact way of writing very large or small numbers. It enables you to compare the sizes of two quantities at a glance; rather than having to count all the zeroes, you just have to look at the powers of ten.

Numbers in scientific notation can be entered into a calculator using the button marked E, EE or EXP. This button can be read as ‘times ten to the power of …’. For example, to enter 1.46×10^{21} you key in 1.46, next press the E button and then key in 21. The calculator will probably display something like $1.46\ ^{21}$ or 1.46 E 21 — most displays do not show the 10.

Beware of two common pitfalls. First, if you key in $1.46 \times 10 \times$ E 21 you have actually entered a number that is ten times too big because the calculator interprets it as $1.45 \times 10 \times 10^{21} = 1.46 \times 10^{22}$. Second, take care how you copy down numbers from a calculator display. If you write down the displayed number as 1.46^{21} you have actually written $1.46 \times 1.46 \times \ldots$ (21 times altogether) which comes to a bit over 2827.5 and is certainly not the same as 1.46×10^{21}.

To enter a power of ten with a negative exponent, you need to use the +/– button directly before or after entering the exponent. For example, 1.67×10^{-27} can be entered by keying in 1.67 E +/– 27 or 1.67 E 27 +/–. The +/– button changes the sign of a number; pressing it twice gets you back to the original sign.

Avoid the temptation to use the arithmetic minus button when entering negative exponents, as the calculator will interpret this as subtracting one number from the previous one. For example, if you type in 1.67 E – 27 a calculator will interpret this as $1.67 \times 10^{0} - 27$, i.e. $1.67 - 27$ which comes to -25.33 and is not what was intended.

QUESTION 6.6

Write the following quantities in scientific notation.

(i) 150 000 000 000 m (the mean Earth–Sun distance)

(ii) 0.000 000 000 000 000 000 160 C (the charge of a proton).

QUESTION 6.7

Use a calculator to evaluate each of the following expressions.

(i) $4.57 \times 10^{14} \times 6.63 \times 10^{-34}$

(ii) $3.00 \times 10^{8}/4.86 \times 10^{-7}$.

6.1.5 Logarithms

The numbers 1, 10, 100, 1000 etc, and 0.1 0.01, 0.001 etc, can all be expressed as whole number powers of ten: 10^0, 10^1, 10^2, 10^3 etc. and 10^{-1}, 10^{-2}, 10^{-3} etc. Fractional powers of ten produce other numbers: for example, $10^{0.5} = 3.162\ldots$. *Any* positive number can be expressed as a power of ten, and the powers can in principle be found by reading from the *graph* in Figure 6.1: if you want to express some number y as a power of ten, then read the corresponding value of x from the graph.

The power to which ten must be raised to produce a given value of y is called the **logarithm** to base ten, or base-ten logarithm, of y, also known as the *common logarithm* of y or, often, just 'the logarithm' or 'the log' of y. In symbols,

$$y = 10^x \quad \text{or} \quad x = \log_{10}(y) \tag{6.12}$$

The symbols $\log(y)$ and $\lg(y)$ are also used in place of $\log_{10}(y)$. (In principle any number can be used as the base for a system of logarithms e.g. if $y = 2^x$, $x = \log_2(y)$. However, the only number apart from 10 that is commonly used for logs is the number e = 2.718… which has special mathematical properties. However, logs to base e, or so-called *natural logarithms* do not feature in S282 or S283.)

The logarithms of whole-number powers of ten can be written down quite easily. For example, 1000 can be written as 10^3, so $\log(1000) = 3$. Likewise, 0.01 can be written as 10^{-2}, so $\log(0.01) = -2$.

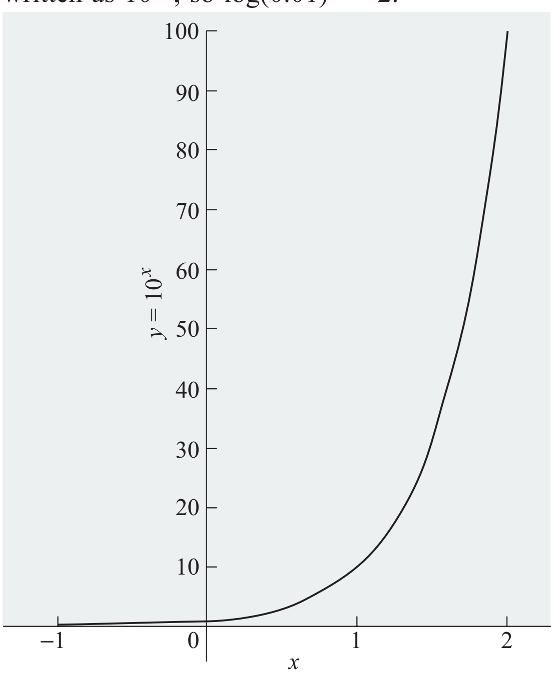


Figure 6.1 A graph of $y = 10^x$ plotted against x.

The logarithm of any positive number can be found using the LOG button on a calculator. Key in the number then press log. Note that negative numbers do not have logarithms, since raising ten to any positive or negative power produces a positive result. Note too that all numbers greater than 1 have positive logarithms while numbers between one and zero have negative logarithms. Since $10^0 = 1$, $\log_{10}(1) = 0$.

QUESTION 6.8

Without using a calculator, write down the logarithms to base ten of the following numbers.

(i) 10 000 (ii) 0.000 01 (iii) 1

QUESTION 6.9

Use a calculator to find the base-ten logarithms of the following numbers.

(i) 2 (ii) 3.172 277 (iii) 0.625

QUESTION 6.10

Find the value of x for each of the following values of 10^x.

(i) 2 (ii) 3.172 277 (iii) 0.625

The process of finding the common logarithm of a number can be reversed. As Equation 6.12 shows, if we take the base-ten log of some number y to get another number x, then finding 10^x gets us back to our original number y. The number y is the base-ten antilogarithm or **antilog** of x:

$$x = \log_{10}(y),\ y = \text{antilog}_{10}(x) \tag{6.13}$$

The base-ten antilogs of whole numbers can be found quite easily by working out the appropriate power of ten. For example,

$$\text{antilog}_{10}(3) = 10^3 = 1000$$

$$\text{antilog}_{10}(-2) = 10^{-2} = 0.01$$

$$\text{antilog}_{10}(0) = 10^0 = 1$$

For other numbers, antilogs can be found on a calculator either by using the y^x button to raise ten to the power of that number, or by keying in the number then pressing the INV (or SHIFT) button followed by the LOG button. For example, to find the antilog of 1.2, either key in 1.2 then press INV and LOG to get the answer 15.848…, or key in 10, press y^x and key in 1.2 to get 15.848….

QUESTION 6.11

By calculating 10^x with the y^x button and/or by using the INV and LOG buttons use a calculator to find $\text{antilog}_{10}(x)$ for each of the following values of x.

(i) 0.5 (ii) 1.23 (iii) −1.23

6.2 Precision

6.2.1 Significant figures

The number of **significant figures** in a quantity is, simply, the number of figures that give meaningful information about its size and precision. For example, if you measure the length of a table to the nearest cm (0.01 m) and find that it was 1.72 m, then the quantity 1.72 m has three significant figures. All three figures carry some meaning about the size of the quantity, and the final '2' indicates that the length really is 1.72 m and not 1.71 m or 1.73 m.

The length written as 1.72 m has two **decimal places** (two figures after the decimal point). The number of decimal places changes according to how a quantity is written but the number of significant figures does not. You could for example write the length 1.72 m as 172 cm (no decimal places, 3 significant figures) or as 0.00172 km (five decimal places, three significant figures).

Zeroes at the beginning of a number are *never* significant. The quantity 001.72 m is exactly the same as 1.72 m and 0.00172 km. After the decimal point, zeroes are important because they give the number its 'place' but they are not significant in any other way.

Zeroes at the end of a number are a bit more tricky. After a decimal point they *are* significant. For example if you measure the distance to the corner shop to the nearest 10 m (0.01 km) and find it is 1.40 km rather than 1.39 km or 1.41 km then the zero is conveying some useful information. If you wrote that the distance was 1.4 km that would mean that the length was nearer to 1.4 km than 1.3 km or 1.5 km — in other words, that you had only measured the length to the nearest hundred metres (0.1 km). But if for some reason you wanted to express the length in metres, you might write 1400 m. The second zero is important because it gives the number its 'place', but it implies that you have measured the distance to the nearest metre and you know that it is 1400 m not 1399 m or 1401 m, so it is misleading and is certainly not significant. To avoid giving the impression that a quantity has been measured more precisely than it actually has, it is best to use *scientific notation* and write the length as 1.40×10^3 m then there is no ambiguity.

QUESTION 6.12

How many significant figures are there in each of the following numbers?

(i) 1.6 (ii) 0.016 (iii) 001.6 (iv) 1.60 (v) 1600

6.2.2 Significant figures in calculations

When two or more quantities are multiplied or divided by one another, the final result cannot be any more precisely-known than the *least* precise quantity used in the calculation. This means that the answer can only have as many significant figures as the least precise quantity. So in any calculation you should always round the *final* answer to a suitable number of significant figures and avoid the temptation to write down the whole calculator display. The following example illustrates this in detail, and the rest of the examples in this booklet, and our answers to the questions, provide many further examples of correct numbers of 'sig figs'.

EXAMPLE 6.3

Suppose you measure the length and width of a table to be 1.72 m and 0.94 m. What is its area?

On a calculator,

$$\text{area} = \text{length} \times \text{width} = 1.72\,\text{m} \times 0.94\,\text{m} = 1.6168\,\text{m}^2$$

But the length and width have only been measured to the nearest 0.01 m so they could be as much as 0.005 m away in either direction. They could be large as 1.725 m and 0.945 m, which would give a calculator display of 1.620 125. Or they could be as small as 1.715 m and 0.935 m which would give a calculator display of 1.603 525. Most of the figures after the decimal point are meaningless — they are not significant.

The least precise value used in the calculation is the width, 0.94 m, which has two significant figures. Rounding any of the calculator displays to two figures produces an answer 1.6 m^2, which makes sense because it is compatible with the possible range of lengths and widths and does not imply that the area can be known any more precisely.

Whole numbers arising from the mathematics of a situation rather than from physical measurement really are exact whole numbers. For example, the *circumference* of a circle with radius r is $2\pi r$ and the '2' is exactly 2, even if you measure it to a huge (infinite) number of decimal places. So it does not limit the precision to which the circumference can be calculated. Similarly, when a number is written in scientific notation the '10' raised to a power is exactly 10 and does not reduce the precision of the overall number. For example, 1.40×10^3 m has three significant figures.

EXAMPLE 6.4

A meteorite of mass 1.65 kg travels at a speed of 2.3 km s^{-1}. What is its *kinetic energy* E_k? ($E_\text{k} = \frac{1}{2}mv^2$) How many figures in the answer are significant?
On a calculator,

$$E_\text{k} = \frac{1.65\,\text{kg} \times (2.3 \times 10^3\ \text{m}\,\text{s}^{-1})^2}{2} = 4364\,250\,\text{J or } 4.364\,25 \times 10^6\ \text{J}$$

As the speed is only known with a precision of 2 significant figures, only two figures of the answer are significant so it should be rounded to 4.4×10^6 J.

If the figure immediately following the final significant one is five or greater, as in Example 6.4, then the preceding value should be rounded up. Sometimes this has a knock-on effect. For example suppose you calculate an answer as 3.987 and you know that only two figures are significant. The second significant figure here is the '9' after the decimal point, and the one following it is '8' so the '9' must be rounded up which in turn means that the '3; becomes a '4. The final result should be written 4.0 not just 4, because the zero after the point *is* significant.

QUESTION 6.13

A space probe (e.g. Apollo 12) takes 33 hours to travel a distance of 241×10^6 m from the Earth to the Moon. What is its average speed? How many figures in the answer are significant? (Speed = distance/time, 1 hour = 3600 s.)

6.2.3 Orders of magnitude

Sometimes when doing scientific calculations we are not interested in the precise answer but just need to know its **order of magnitude** — in other words, the nearest power of ten. For example, the proton's charge is, $e = 1.60 \times 10^{-19}$ C to three *significant figures* but when expressed as an order of magnitude is simply 10^{-19} C; we can say that it is 'of the order of 10^{-19} C' (or 'of order 10^{-19} C'). In symbols

$$e \sim 10^{-19}\ \mathrm{C}$$

The radius, r, of the Sun is 6.96×10^7 m, and to the nearest order of magnitude this is 10^8 m.

$$r \sim 10^8\ \mathrm{m}$$

Notice that the value for the proton charge is rounded down while that of the Sun's radius is rounded up.

QUESTION 6.14

Express the following quantities to the nearest order of magnitude.

(i) Boltzmann constant, $k = 1.38 \times 10^{-23}\ \mathrm{J\,K^{-1}}$

(ii) The number 4π

(iii) The Sun's mass, 1.99×10^{30} kg

(iv) The distance of Jupiter from the Sun, 7.78×10^{11} m

QUESTION 6.15

In a vacuum, light travels at $3.00 \times 10^8\ \mathrm{m\,s^{-1}}$. There are 3.1536×10^7 seconds in one year. To the nearest order of magnitude how far does light travel through space in one year? (Use distance = speed × time, and do not use a calculator!)

6.3 Algebra and equations

6.3.1 Equations

An **equation** is a mathematical statement that two quantities are *equal*, that is, exactly the same as one another. An equation involves two different combinations of numbers and quantities, linked by an equals sign, =. Generally we use equations to relate two or more physical quantities to one another, and the quantities are represented by letters as this makes it easier to manipulate the equations. The general process of using and manipulating equations involving symbols is called **algebra**.

For example, *speed* is conventionally represented by u, v or (particularly for electromagnetic radiation) by c, distance by x, s or d, and time interval by t. These three quantities are related by an equation:

$$\text{distance travelled} = \text{speed} \times \text{time interval} \tag{6.14a}$$

or

$$x = ut \tag{6.14b}$$

In Equation 6.14, distance, x, is the **subject** of the equation; the equation shows how it can be calculated by substituting values for the quantities on the right-hand side.

Any equation can be rearranged to make another quantity the subject. In rearranging an equation, the crucial thing to remember is that both sides must always be equal. To maintain this situation, you must always do *exactly* the same thing to both sides of an equation.

When rearranging an equation, it can sometimes be helpful if you start off swapping the left- and right-hand sides of an equation so that the part with the quantity you are interested in is on the left-hand side. (You don't *have* to do this, but it can make it easier to see where you are going. And if you *do* this, be careful *only* to exchange left and right sides — don't change anything else!) Next you need to 'undo' all the things that involve that quantity and hence get it on its own. If the quantity is multiplied by something, then you can 'undo' that by division. For example, to make speed, u, the subject of Equation 6.14, you can write

$$ut = x$$

then divide both sides by t:

$$ut/t = x/t$$

On the left-hand side ut/t is just the same as u (the two 't's cancel) so we have

$$u = x/t \tag{6.15}$$

You can undo division by multiplication and similarly you can undo addition by subtraction and vice versa.

EXAMPLE 6.6

An object's acceleration, a, can be calculated from its initial speed, u, final speed, v, and the time interval, t, taken for the speed to change from u to v:

$$a = (v - u)/t \tag{6.16}$$

Given the initial speed, acceleration, and time interval, how can you calculate the final speed? To make v the subject, first write

EXAMPLE 6.5

An object's acceleration, a, can be calculated from its initial speed, u, final speed, v, and the time interval, t, taken for the speed to change from u to v:

$$a = (v - u)/t \tag{6.16}$$

Given the initial speed, acceleration, and time interval, how can you calculate the final speed? To make v the subject, first write

$$(v - u)/t = a$$

Then multiply both sides by t:

$$t(v - u)/t = at$$

so $(v - u) = at$

and add u to both sides:

$$u + v - u = u + at$$

thus $v = u + at$ (6.17)

Notice the order in which we do and undo the various steps. In Equation 6.16 we would first subtract u from v then divide by t. We have to undo the steps in the reverse order, so we undid the division first, because the whole of $(v - u)$ was divided by t, and then we were able to undo the subtraction of u from v.

To undo a square, take a *square root* and to undo a *reciprocal* take another reciprocal. The key thing is always to think of *doing the same thing to both sides* as the following examples show.

EXAMPLE 6.6

The *kinetic energy* E_k of an object mass m moving at speed v is given by

$$E_k = \frac{mv^2}{2} \tag{6.18}$$

How can you calculate speed from given values of E_k and m?

You need to make v the subject of the equation. Write

$$\frac{mv^2}{2} = E_k$$

To get v on its own, first get v^2 on its own: multiply both sides by 2 and divide by m:

$$\frac{2mv^2}{2m} = \frac{2E_k}{m}$$

so $v^2 = \dfrac{2E_k}{m}$

Then take the square root of both sides:

$$\sqrt{v^2} = \sqrt{\frac{2E_k}{m}}$$

therefore $$v = \sqrt{\frac{2E_\text{k}}{m}} \tag{6.19}$$

Notice that the square root covers *everything* on the right-hand side.

EXAMPLE 6.7

The *density* ρ of a substance is related to the mass m and volume V of a sample.

$$\rho = \frac{m}{V} \tag{6.20}$$

How can you calculate the volume occupied by a given mass of a substance if you know its density?

To make V the subject of the equation, write

$$\frac{m}{V} = \rho$$

To get V on top, take the *reciprocal* of both sides:

$$\frac{1}{(m/V)} = \frac{1}{\rho}$$

so $$\frac{V}{m} = \frac{1}{\rho}$$

then multiply both sides by m:

$$\frac{mV}{m} = \frac{m}{\rho}$$

thus

$$V = \frac{m}{\rho} \tag{6.21}$$

Alternatively, you might prefer a slightly different, but equivalent, route. To start, multiply both sides of Equation 6.20 by V

$$\frac{Vm}{V} = V\rho$$

thus $m = V\rho$

Then divide both sides by ρ:

$$\frac{m}{\rho} = V$$

which is exactly the same as Equation 6.21 when you interchange the left- and right-hand sides.

QUESTION 6.16

Force F, mass m and acceleration a are related by the Equation $F = ma$. Rearrange this equation to make a the subject.

QUESTION 6.17

The *gravitational force*, F, between two masses M and m with their centres separated by a distance r is given by the equation $F = GMm/r^2$ where G is the gravitational constant. Rearrange this equation to make r the subject.

6.3.2 Combining equations

It is often useful to combine two equations to produce a third that allows you to calculate a particular quantity. Sometimes this can allow you to tackle a problem that at first seems impossible, as the next example shows.

EXAMPLE 6.8

An apple falls from a tree through a height Δh. As it falls it loses *gravitational potential energy*

$$\Delta E_g = mg\,\Delta h$$

and gains kinetic energy

$$\Delta E_k = \frac{mv^2}{2}$$

Without knowing the mass of the apple, use these equations to calculate its speed v after it has fallen through a given height.

The key thing here is to note that the loss in gravitational energy is equal to the gain in kinetic energy. You could guess a mass, work out the loss in E_g then set that equal to the gain in E_k, then use that to find the speed, which would involve dividing by the mass that you had guessed…. It is much neater to do some algebra first and write

$$\frac{mv^2}{2} = mg\,\Delta h$$

Dividing both sides by m we have

$$\frac{v^2}{2} = g\,\Delta h$$

and so we can multiply both sides by 2 and take the *square root* to make v the subject:

$$v = \sqrt{2g\,\Delta h}$$

Sometimes the **elimination** of the unwanted quantity is even less obvious. For example, we might want to calculate the photon energy of some *electromagnetic radiation* whose *wavelength* λ we know. The *photon energy*, E_{ph}, is related to the *frequency*, f, of the radiation

$$E_{ph} = hf \tag{6.22}$$

where h is the Planck constant. The frequency is in turn related to the wavelength:

$$c = f\lambda \tag{6.23}$$

where c is the wave speed of electromagnetic radiation. You can of course calculate f using Equation 6.23 then put that value into Equation 6.22 to find the photon energy, but it is much neater to use algebra to produce an equation for the quantity you want and thus avoid any unnecessary calculation. To eliminate the unwanted quantity, first make f the subject of Equation 6.23:

$$f = c/\lambda$$

then because c/λ is exactly the same as f, you can put c/λ instead of f in Equation 6.22

$$E_{ph} = hc/\lambda \tag{6.24}$$

This process of replacing one symbol by another combination is called **substitution**. Notice the steps involved. First make the unwanted quantity the *subject* of an equation involving things that you know. Then use that to replace the unwanted quantity in the other equation.

An alternative way to eliminate the unwanted quantity is to make it the subject of *both* equations:

$$f = c/\lambda$$

and, from Equation 6.22

$$f = E_{ph}/h$$

Then because the right-hand sides are both equal to the unwanted quantity, they must also be equal to each other. They can therefore be linked by an equals sign to produce a new equation without the unwanted quantity:

$$E_{ph}/h = c/\lambda$$

This can be rearranged to get the desired subject as before:

$$E_{ph} = hc/\lambda$$

These methods are essentially the same, but substitution is usually less long-winded.

QUESTION 6.18

The distance around the circumference of a circle radius r is $2\pi r$. Use $v = x/t$ to derive an expression for the speed, v, of a planet that takes a time T to travel once around a circular orbit of radius r.

QUESTION 6.19

When a mass m of a substance is heated through a temperature rise ΔT, the change in its *internal energy*, Δq, is given by $\Delta q = mc\,\Delta T$ where c is the *specific heat capacity* of the substance. The *density* ρ of a substance is related to the mass m and volume V of a sample: $\rho = m/V$. By substituting a suitable expression for m, produce an equation that shows how to calculate Δq when a volume V of a substance with a given ρ and c is heated through ΔT.

6.4 Proportionality

6.4.1 Direct proportionality

If one quantity is **directly proportional** (or just **proportional**) to another, whenever one is multiplied or divided by a given factor then so is the other. For example, for travel at a constant speed, distance covered is proportional to the time taken: if you travel for three times as long then you cover three times the distance. Similarly, the *acceleration* of a given object is proportional to the net *force* acting on it: if you halve the force you also halve the acceleration. One quantity is related to the other by a constant factor, called a **constant of proportionality**. If some quantity y is proportional to another quantity x we can write this in various ways:

$$y = kx \tag{6.25a}$$

or

$$y/x = k \tag{6.25b}$$

where k is the constant of proportionality. Plotting a *graph* of y against x produces a straight line through the origin, with *gradient* k. Another way to write Equation 6.25 is

$$y \propto x \tag{6.25c}$$

where the sign $\propto$ is read 'is (directly) proportional to'.

In our example of travel at constant speed, u, it is the speed itself that is the constant of proportionality because distance x is related to time t via the equation

$$x = ut$$

Some equations relating physical quantities include several proportionality relationships. For example, when a mass m of a substance is heated through a temperature rise ΔT, the change in its *internal energy*, Δq, is given by

$$\Delta q = mc\,\Delta T \tag{6.26}$$

where c is the s*pecific heat capacity* of the substance. From Equation 6.26 we can identify two proportional relationships that describe how Δq behaves for a given substance (i.e. a given value of c):

$$\Delta q \propto m$$

that is if you multiply or divide the mass by a given factor then you multiply or divide Δq by the same factor. Also

$$\Delta q \propto \Delta T$$

Proportional relationships can also involve quantities with *exponents*. For example, the kinetic energy E_k of an object mass m moving at speed v is given by

$$E_k = \frac{mv^2}{2} \tag{6.27}$$

so we can write

$$E_k \propto v^2$$

If we multiply the speed v by 2, then v^2 is multiplied by 4 so E_k is also multiplied by 4 (= 2^2), and if we multiply the speed by 3 then v is multiplied by 9 (= 3^2) and so

also is E_k. Whatever factor we use to multiply or divide v, then E_k is multiplied or divided by the square of that factor.

Note that a proportional relationship always tells us how one quantity will *change* as another *changes*, so both sides of the relationship must be things that can vary. It is meaningless to write that a quantity is proportional to a constant, because a constant, by definition, does not change. So for example we can say that *rest energy* E_0 is proportional to mass m from the equation $E_0 = mc^2$, but we *cannot* write that $E_0 \propto c^2$ because c, the speed of electromagnetic radiation in a vacuum, is fixed.

QUESTION 6.20

The pressure p exerted by n molecules of gas at absolute temperature T is given by $p = nkT$ where k is the Boltzmann constant. Write proportional relationships to show how p depends on other quantities in the equation.

6.4.2 Inverse proportionality

Sometimes multiplying one quantity by a given factor results in another being divided by the same factor. For example, if you double the *frequency* of waves that are travelling at a fixed speed, then you halve their *wavelength*. And if you multiply your *speed* by three, then you cover a fixed distance in one-third of the time. In these example one quantity is **inversely proportional** to the other. (Note the correct term: there is no such thing as 'indirectly proportional'.) If some quantity y is inversely proportional to some quantity x then we can write

$$y = k/x \tag{6.28a}$$

or

$$yx = k \tag{6.28b}$$

where k is a constant of proportionality.

There is no separate sign meaning 'is inversely proportional to' so we write

$$y \propto 1/x \tag{6.28c}$$

i.e. y is directly proportional to the reciprocal of x, usually read as 'y is inversely proportional to x'. As with direct proportionality, inverse proportionality can involve quantities with *exponents*.

QUESTION 6.21

Write proportionality relationships between x and y in each of the following examples.

(i) $y = 27A/x$ where A is a constant

(ii) $y = 4\pi x^2$

(iii) $y = GMm/x^2$

6.4.3 Combining proportional relationships

Proportional relationships can be combined. For example, when Newton developed his *law of gravitation* he stated that the magnitude of the gravitational *force*, F_g, between two masses m_1 and m_2, with their centres separated by a distance r, was directly proportional to each of the masses and inversely proportional to the square of their separation:

$$F_g \propto m_1$$

$$F_g \propto m_2$$

$$F_g \propto \frac{1}{r^2}$$

We can combine these expressions by multiplying all the right-hand sides together:

$$F_g \propto \frac{m_1 m_2}{r^2}$$

Here the *constant of proportionality* is G, the gravitational constant.

$$F_g = \frac{G m_1 m_2}{r^2}$$

EXAMPLE 6.9

The lifetime, t, of a star similar to our Sun is proportional to its mass M and inversely proportional to its output *power*, or luminosity, L:

$$t \propto M/L$$

Its luminosity in turn depends on its mass:

$$L \propto M^5$$

Combine these proportional relationships into one that describes how a Sun-like star's lifetime depends on its mass.

It is possible to go round in circles when dealing with situations like this, as it looks as though everything is proportional to everything else. Things become more manageable if we turn the proportionalities into equations, and for example write

$$t = kM/L$$

$$L = KM^5$$

where k and K are some *constants of proportionality* whose actual values we don't really care about. Now we can substitute for L in our first equation

$$t = \frac{kM}{KM^5}$$

As $M/M^5 = M^{-4}$ we can write

$$t = \frac{k}{K} M^{-4}$$

but as we are only interested in how t varies with M we can drop the K and k and write another proportional relationship:

$$t \propto M^{-4}$$

QUESTION 6.22

A star's output *power*, known as its luminosity, L, depends on its surface *temperature* T and on its surface area A. The surface area A in turn depends on the star's radius r.

$$L \propto T^4,\ L \propto A \text{ and } A \propto r^2$$

Combine these proportional relationships to get a single proportional relationship that described how a star's luminosity L depends on T and r.

6.4.4 Using proportionality in problems

Situations sometimes arise when we need to use a proportional relationship in a problem. For example, stars similar to our Sun have lifetimes t that depend on their mass, M, such that $t \propto M^{-4}$. The Sun's lifetime is estimated to be 10 Ga. We can ask what would be the lifetime of a similar star whose mass is twice that of the Sun.

There are several ways to tackle this. All of them boil down to the same thing but some are neater than others. In all of them, start by writing down an equation:

$$t = KM^{-4} \tag{6.29}$$

where K is a *constant of proportionality* whose value we don't know.

A long-winded way to tackle the problem would be to make K the *subject* of Equation 6.29,

$$K = t/M^{-4} = tM^4 \tag{6.30}$$

and put in values for the Sun's mass and lifetime to calculate K. Then use this same K in Equation 6.29, putting in a mass that is twice that of the Sun.

A much neater way is to use symbols t_1 and M_1 to represent the Sun's lifetime and mass, and t_2 and M_2 to represent those of the other star. We can then write:

$$t_1 = KM_1^{-4} \tag{6.31a}$$

and $$t_2 = KM_2^{-4} \tag{6.31b}$$

Rearranging gives

$$K = t_1 M_1^4$$

and $$K = t_2 M_2^4$$

As both right-hand sides are equal to K we can write

$$t_1 M_1^4 = t_2 M_2^4 \tag{6.32}$$

Dividing both sides by M_2^4 we get

$$t_2 = t_1 \left(\frac{M_1}{M_2}\right)^4 \tag{6.33}$$

and because $M_2 = 2M_1$, we can write

$$t_2 = t_1 \left(\frac{M_1}{2M_1}\right)^4 = \frac{t_1}{2^4} \tag{6.34}$$

so we can solve the problem without having to calculate K and without having to know the actual masses of the Sun and the other star.

There is a variation on this method, which gets to the same result using different steps. Starting with the two versions of Equation 6.31, we can divide the left-hand side of Equation 6.31b by the left-hand side of Equation 6.31a to get t_2/t_1. Since we are dealing with *equations*, we must do exactly the same to both sides and so we must also divide the right-hand side of Equation 6.31b by t_1. But since $t_1 = KM_1^{-4}$, we can divide the right-hand side by KM_1^{-4} to get

$$\frac{t_2}{t_1} = \frac{KM_2^{-4}}{KM_1^{-4}} = \left(\frac{M_2}{M_1}\right)^{-4} = \left(\frac{M_1}{M_2}\right)^{4}$$

which can be rearranged to get Equation 6.34 as before.

QUESTION 6.23

The orbits of planets around the Sun are described by the relationship $t^2 \propto r^3$ where t is the time to travel once around the orbit and r is the radius of the orbit. The Earth takes one year to travel once around its orbit. The planet Jupiter has an orbit whose radius is 5.2 times that of the Earth's. How long does it take Jupiter to travel once around its orbit?

6.5 Circles, angles and trigonometry

6.5.1 Circles and spheres

Circles and spheres are important shapes in astronomy and planetary science. Planets in the Solar System move in approximately circular orbits, as do many stars in their orbits in the Galaxy, and stars and planets themselves are approximately spherical.

The diameter, D, of a circle is twice its radius, r, (Figure 6.2) and the **circumference**, C, (the distance around the edge) is related to the radius and diameter

$$C = \pi D = 2\pi r \tag{6.35}$$

the number represented by π (Greek letter pi) is equal to 3.141 592 654 (to 9 *decimal places*) and is generally stored on a calculator — look for the π button.

The area, A, of a circle is related to its radius:

$$A_{\text{circ}} = \pi r^2 \tag{6.36}$$

Putting $D = 2r$ in Equation 6.36 we can also see how the area is related to the diameter:

$$A_{\text{circ}} = \pi(D/2)^2 = \pi D^2/4 \tag{6.37}$$

The surface area, A, of a sphere is also related to its radius:

$$A_{\text{sph}} = 4\pi r^2 \tag{6.38}$$

and so is its volume, V:

$$V_{\text{sph}} = 4\pi r^3/3 \tag{6.39}$$

Notice that the areas of a circle and a sphere have dimensions of length × length (that is r^2) and so they have SI units of m^2, and the volume of a sphere has dimensions of $length^3$ and SI units m^3; the other numbers in the equations are dimensionless.

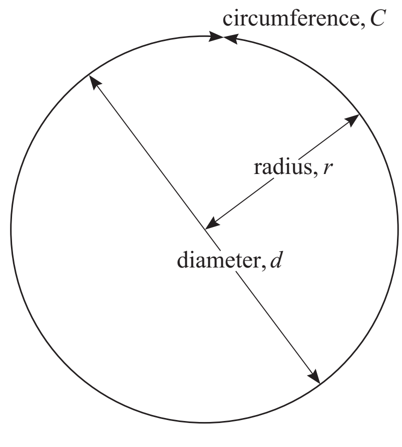


Figure 6.2 A circle.

QUESTION 6.24

Derive equations for (i) the surface area and (ii) the volume of a sphere in terms of its diameter.

QUESTION 6.25

The Earth moves around the Sun in an approximately circular orbit of radius 1.50×10^{11} m. What is the distance around the orbit?

QUESTION 6.26

The Sun is a sphere with radius 6.96×10^{8} m. What are (i) its surface area and (ii) its volume?

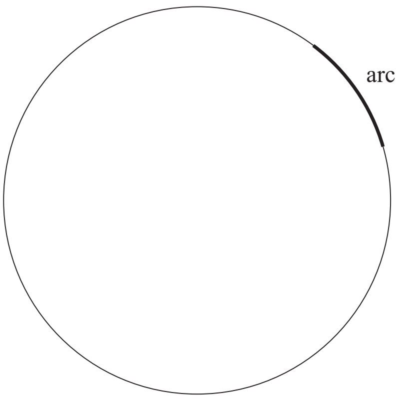


Figure 6.3 An arc of a circle.

6.5.2 Angles

In astronomy and planetary science we are often concerned with the angle between two different directions, for example the directions to either 'edge' of a planet or a nebula. Angles are often measured in **degrees of arc** (often just called **degrees**). The term **arc** refers to a part of the curved edge of a circle (Figure 6.3). There are 360 degrees of arc (360 deg or 360°) in one complete turn. A quarter-turn is 90°, and is called a **right angle**. Figure 6.4 shows some angles measured in degrees. Notice the way they are marked and labelled.

For expressing small angles, the degree of arc is divided into 60 **minutes of arc** (60 **arcmin**, 60′) and a minute of arc is further divided in 60 **seconds of arc** (60 **arcsec**, 60″). Angles in astronomy tend to be very small so the arcsec is a common unit. You will probably not often need to convert between the two ways of expressing angles measured in degrees, but these two examples show how it is done.

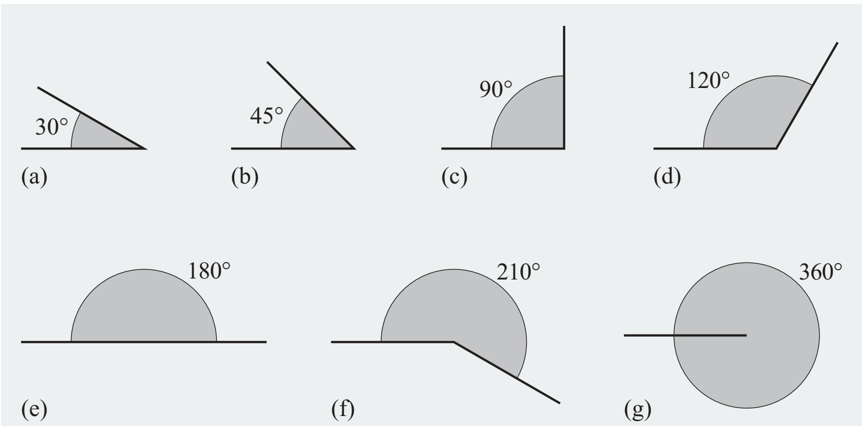


Figure 6.4 Some angles measured in degrees of arc.

EXAMPLE 6.10

A certain angle is written as 2° 24′ 30″. Express this angle as a decimal number of degrees.

Start from the small end: 30″ = 0.50′ so the angle is 2° 24.50′.

Now 24.50′ = (24.50/60) degrees = 0.4083° so the measured angle is 2.4083°.

EXAMPLE 6.11

A certain angle is written as 3.4567°. Express this in degrees, minutes and seconds of arc.

Start from the large end: the angle is 3 degrees and some number of minutes and seconds. $0.4567 \times 60 = 27.402$ so there are 27 arcmin and some number of seconds. $0.402 \times 60 = 24.12$ so there are 24.12 arcsec (24 to the nearest whole number). Therefore the angle is 3° 27′ 24″.

In some circumstances the degree/arcmin/arcsec system of units for measuring angles is not the most convenient, and angles are expressed in **radians** instead. One radian is defined as shown in Figure 6.5: it is the angle between two lines drawn from the centre of a circle such that the length of *arc* between the ends is equal to the radius.

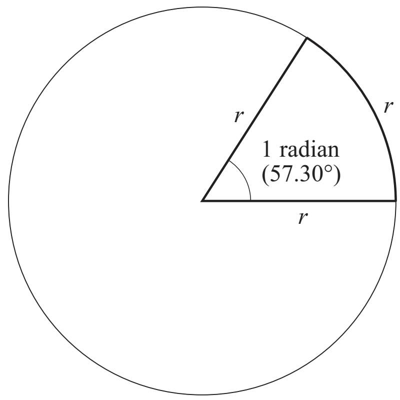


Figure 6.5 Defining the radian.

From this definition, we generalize to any angle:

$$\text{angle measured in radians} = \text{arc length/radius} \tag{6.40a}$$

In symbols

$$\theta/\text{rad} = l/r \tag{6.40b}$$

where θ is the Greek letter theta, often used to represent angles, and l is the length of the arc. Notice that the equation includes the SI unit of radians, often abbreviated to rad.

A complete turn covers an arc length of $2\pi r$ (the *circumference* of the circle) so this corresponds to 2π radians. We can therefore write

$$360° = 2\pi \text{ radians} \tag{6.41}$$

so $$1 \text{ radian} = 360°/2\pi = 57.30° \tag{6.42}$$

and $$1° = 2\pi \text{ radians}/360 = 0.017\,45 \text{ radians} \tag{6.43}$$

Angles measured in radians are sometimes expressed using fractions or multiples of the number π, but not necessarily so. These examples show how to convert between radians and degrees.

EXAMPLE 6.12

What is a right angle expressed in radians?

A right angle is one-quarter of a complete revolution, so it can be written as 2π rad/4 $= \pi/2$ radians = 1.571 radians. You could get the same answer using Equation 6.43, namely $90 \times 0.017\,45$ radians = 1.571 radians.

EXAMPLE 6.13

A certain angle is written as $\pi/6$ radians. What is this in degrees?

From Equation 6.41, $\pi/6$ radians must correspond to one-twelfth of a complete turn, that is 30°. Alternatively, from Equation 6.42, we have $(360°/2\pi) \times \pi/6 = 30°$.

EXAMPLE 6.14

What is an angle of 1 arcmin expressed in radians?

1 arcmin = 1°/60, and from Equation 6.43, we have (2π radians/360) × (1/60) = 2.909×10^{-4} radians.

QUESTION 6.27

What is 60° expressed in radians? Express your answer as a multiple or fraction of π and as a decimal number of radians.

QUESTION 6.28

What is π/4 radians expressed in degrees?

QUESTION 6.29

What is an angle of 1 arcsec expressed in radians?

6.5.3 Right-angled triangles

The branch of maths called **trigonometry** deals with relationships between angles and distances so it is very useful in astronomy and planetary science. Many of the important relationships in trigonometry can be illustrated via right-angled triangles, that is triangles that contain one angle of 90°. Figure 6.6 shows a right-angled triangle. Notice the ‘square’ symbol used to label the right angle. One of the other angles is labelled with the *Greek letter* theta (θ). Since the angles inside *any* triangle always add up to 180°, the unlabelled angle must be equal to $90° - \theta$.

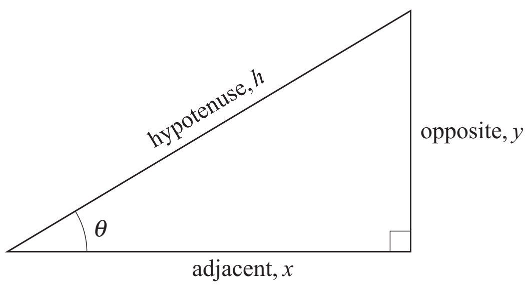


Figure 6.6 A right-angled triangle.

The longest side of a right-angled triangle, opposite the right angle, is called the hypotenuse. The two shorter sides that are opposite and adjacent to (next to) the angle we are interested in (θ) are called the opposite and adjacent sides.

The sides of a right-angled triangle have a useful property, known as **Pythagoras’ theorem**, after the Greek mathematician and philosopher who lived in the 6th century BC. Representing the length of the hypotenuse by h and the other two sides by x and y as shown in Figure 6.6,

$$x^2 + y^2 = h^2 \tag{6.44}$$

If we know the lengths of two sides we can always work out the third. For example, if $x = 3$ m and $y = 4$ m, then $x^2 + y^2 = 9\,\text{m}^2 + 16\,\text{m}^2 = 25\,\text{m}^2$. So $h^2 = 25\,\text{m}^2$ and $h = 5$ m. The numbers don’t always work out so neatly, but Equation 6.44 applies to *all* right-angled triangles. Notice that you need to square each length separately before adding them together, and then find the *square root*.

EXAMPLE 6.15

A certain right-angled triangle has sides $x = 8.5$ cm, $y = 5.0$ cm. How long is its hypotenuse?

From Equation 6.44,

$$h^2 = (8.5\,\text{cm})^2 + (5.0\,\text{cm})^2 = 72.25\,\text{cm}^2 + 25.0\,\text{cm}^2 = 97.25\,\text{cm}^2$$

so $h = \sqrt{97.25\,\text{cm}^2} = 9.9\,\text{cm}$

QUESTION 6.30

The hypotenuse of a certain right-angled triangle is 7.2 cm long. One of the other sides measures 6.5 cm. How long is the third side?

6.5.4 Trigonometric functions

A second useful property of triangles concerns their angles as well as their sides. The relative lengths of the sides of a triangle depend *only* on the angles enclosed. Put another way round, the angles in the triangle depend *only* on the relative lengths of the sides. In a right-angled triangle, the ratios between the sides are given the names: **sine**, **cosine** and **tangent** often abbreviated to sin, cos and tan. For any angle θ in a right-angled triangle (see Figure 6.6)

$$\sin\theta = \text{opposite/hypotenuse} = y/h \qquad (6.45)$$

$$\cos\theta = \text{adjacent/hypotenuse} = x/h \qquad (6.46)$$

$$\tan\theta = \text{opposite/adjacent} = y/x \qquad (6.47)$$

These three trigonometric functions are stored in a calculator. For example, to find the sine of 30°, you need to key in sin 30, or 30 sin (depending on how your calculator operates) and the calculator will display 0.5. Notice that the sin and cosine of any angle are never greater than 1 because the two sides x and y are always less than h, but the tangent of an angle can have *any* value.

EXAMPLE 6.16

A surveyor stands 20 m from a tower and finds that the angle from the top of the tower to the ground is 60°. See Figure 6.7. How tall is the tower?

Compare Figures 6.6 and 6.7. Put distance to tower = x = 20 m, and height = y, then use Equation 6.47:

$$y = x\tan\theta = 20\,\text{m} \times \tan 60° = 35\,\text{m}$$

(First use a calculator to find tan 60°, then multiply by 20.)

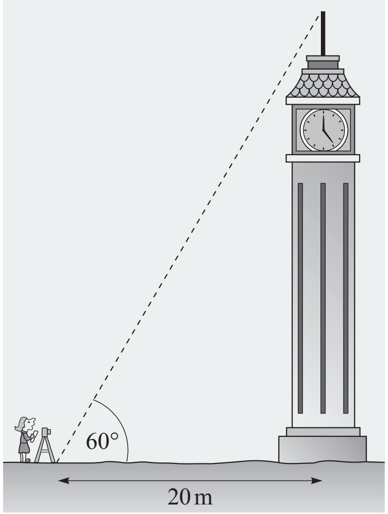


Figure 6.7 Using trigonometry to find the height of a tower.

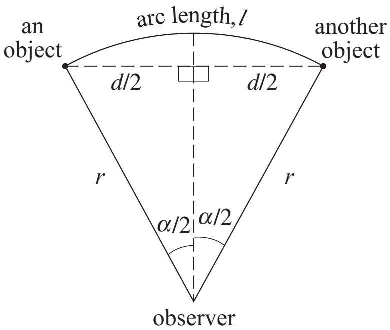


Figure 6.8 Two objects separated by an angle α.

EXAMPLE 6.17

An astronomer observes two objects at a distance r. The angle between the directions to the objects is α. Use Figure 6.8 to derive an equation relating the objects' separation, d, to their distance and the angle α.

Work with one of the right-angled triangles containing the angle $\alpha/2$. The length of the hypotenuse is r and the side we want to know, $d/2$, is opposite the angle $\alpha/2$, so we can use Equation 6.45. Put $\theta = \alpha/2$, $y = d/2$ and $h = r$

$$\sin(\alpha/2) = d/2r$$

multiply by $2r$:

$$d = 2r \sin(\alpha/2) \tag{6.48}$$

Notice the brackets around $\alpha/2$ to ensure that we find the sine of half the angle α (which is *not* the same as finding $\sin\alpha$ then halving the result, so the 2s do *not* cancel out).

QUESTION 6.31

Use one of Equations 6.45, 6.46 or 6.47 to find the distance from the top of the tower to the place where the surveyor is standing in Figure 6.7.

Take care, when obtaining the sine, cosine or tangent of an angle, that the units of the angle match those of the calculating procedure. Most calculators can be switched to receive angles in *radians* or rather than degrees (try pressing a button marked DRG, or consult the instructions for your calculator).

EXAMPLE 6.18

The angle in Figure 6.7 could also be labelled π/3 radians. To work out the height of the tower, use the same method as Example 6.17. Use your calculator to work out π/3 (= 1.047…) then make sure it is in 'radian mode' and press tan to get tan(π/3 rad) = 1.732…. Then multiply by 20 to get the answer 35 m as in Example 6.17.

QUESTION 6.32

Use $\theta = \pi/3$ radians in one of Equations 6.45, 6.46 or 6.47 to find the distance from the top of the tower to the place where the surveyor is standing in Figure 6.7.

6.5.5 Small angles

Astronomers often need to deal with angles that are very small, for example when they observe objects whose size or separation is small compared with their distance. In Figure 6.9, if the angle θ is small, the distance d becomes very similar to the arc length l. So if θ is measured in *radians*, we can write

$$d/r \approx l/r = \theta/\text{rad} \tag{6.49}$$

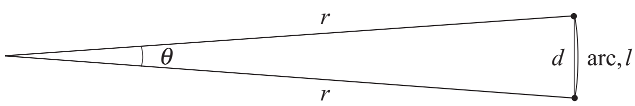


Figure 6.9 Small angle approximation.

This **small angle approximation** enables us to find d very easily using r and the angle θ just by putting θ in radians with no need for trigonometric functions.

The small angle approximation leads to some further useful approximations involving trigonometric functions. From triangle OAC in Figure 6.10, when θ is small

$$\sin\theta = \mathrm{AC}/r \approx l/r = \theta/\mathrm{rad} \tag{6.50}$$

and $\cos\theta = \mathrm{OA}/r \approx r/r = 1$ (6.51a)

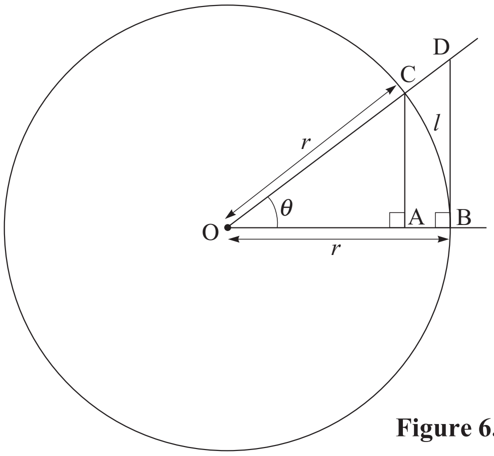


Figure 6.10 Trigonometric functions of the angle θ.

From triangle OBD in Figure 6.10 when θ is small

$$\tan\theta = \mathrm{BD}/r \approx l/r = \theta/\mathrm{rad} \tag{6.52}$$

and $\cos\theta = r/\mathrm{OD} \approx r/r = 1$ (6.51b)

So from Equations 6.50 and 6.52 we have, for small θ

$$\sin\theta \approx \tan\theta \approx \theta/\mathrm{rad} \tag{6.53}$$

The range of angles for which we can use the small angle approximation depends on how precise we want to be. For an angle of 1° (1.745×10^{-2} rad) the results using the approximation are correct to at least three significant figures, and the approximation gets even better for smaller angles.

EXAMPLE 6.19

Viewed from Earth, the Sun has an angular diameter α close to 0.5° (the angle between two lines pointing towards opposite edges of the Sun). Given that its distance from Earth is $r = 1.50 \times 10^{11}$ m, what, approximately, is its actual diameter?

From Equation 6.42, $0.5° = 8.7 \times 10^{-3}$ rad. Then from Equation 6.49, diameter $d = \alpha r = 8.7 \times 10^{-3} \times 1.50 \times 10^{11}\,\mathrm{m} = 1.3 \times 10^{9}\,\mathrm{m}$.

EXAMPLE 6.20

Use results from the small angle approximation to simplify Equation 6.48, which was derived in Example 6.18 for the separation of the two objects in Figure 6.8.

Using Equation 6.53

$$d = 2r\sin(\alpha/2) \approx 2r(\alpha/2)/\mathrm{rad} = r\alpha/\mathrm{rad}$$

From Figure 6.8, $\alpha/\mathrm{rad} = l/r$ so $d = \alpha l$, as expected from Equation 6.49.

QUESTION 6.33

The Moon's diameter is $d = 3.5 \times 10^6$ m. Its angular diameter, viewed from Earth, is approximately 0.5°. What is its distance from Earth?

6.6 Graphs

6.6.1 Plotting and reading graphs

A **graph** illustrates how two quantities are related to one another by displaying points corresponding to pairs of values. The points are often joined by a straight line or a smooth curve. For example, Figure 6.11 is a graph of an object's speed, v, plotted against time, t. In this example, the object was already moving at *speed* $u = 3.0\,\mathrm{m\,s^{-1}}$ when $t = 0.0$ s, and its *acceleration* was $a = 2.0\,\mathrm{m\,s^{-2}}$. Table 6.1 shows the values of v calculated at various times using the equation

$$v = u + at \qquad (6.54)$$

Table 6.1 Data for Figure 6.11.

Time t/s	Speed v/m s^{-1}
0.0	3.0
1.0	5.0
3.0	9.0
4.0	11.0

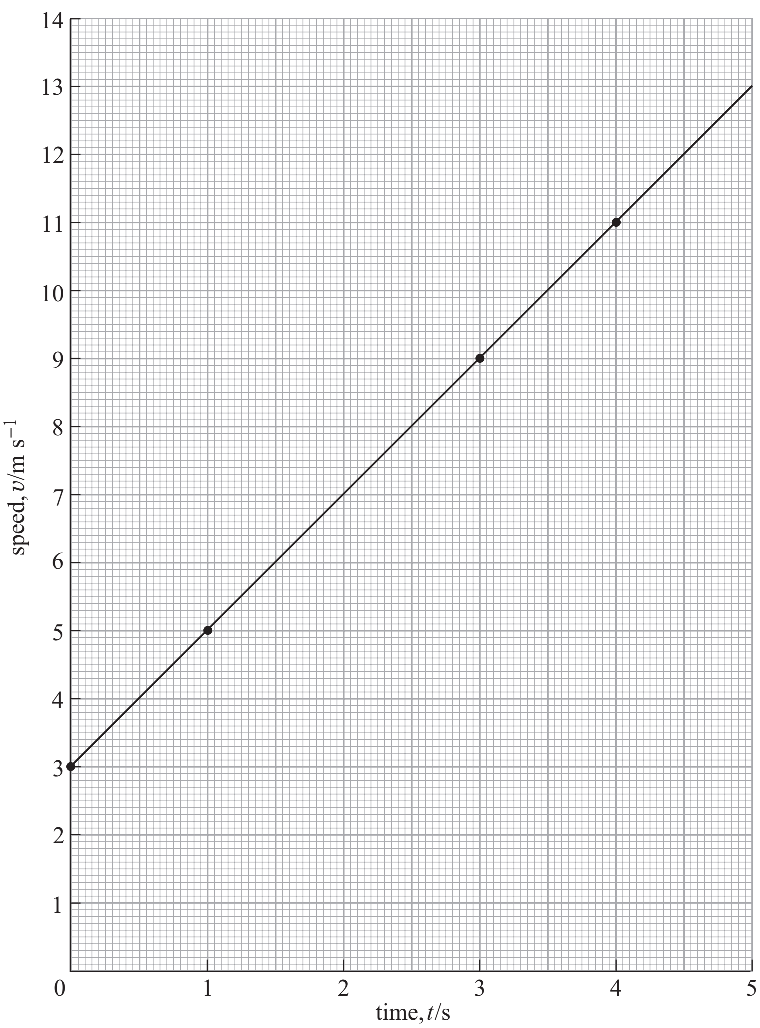


Figure 6.11 A graph of speed against time.

Notice that the graph's **axes** (the lines that show where the numbers on the graph should be plotted) have a **linear scale** — that is, they are marked at equal intervals even though the values in Table 6.1 are not evenly spaced, and are labelled in the form 'quantity/unit'. The axis running across the page is generally called the *x*-axis, and the one running vertically is the *y*-axis, even though the quantities plotted on them quite often have symbols other than *x* and *y*. We often say that values of the quantity on the *y*-axis are 'plotted against' corresponding values of the quantity on the *x*-axis. In Figure 6.11, the four plotted points are marked with dots and, because in this case they all lie on a straight line, are joined with a straight line.

The **origin** of a graph is the point corresponding to zero on both axes, and this is usually drawn at the bottom left of the graph where the axes cross. Sometimes, if we want to plot a small range of values that lie very far from zero, we choose to draw a graph whose axes do *not* start at the origin. Provided the axes are clearly labelled, this is fine. Figure 6.12 gives an example where it would clearly be silly to start with the year zero on the *x*-axis, and if we started at zero on the *y*-axes then all the values would be squashed to the top of the graph making them very hard to read.

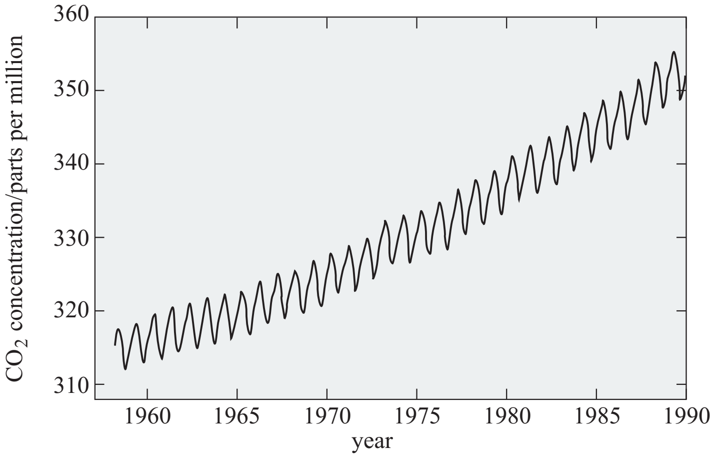


Figure 6.12 A graph showing variations in atmospheric carbon dioxide. An example of a graph that does not include the origin.

In plotting a graph, the convention is usually that the **independent variable** is plotted along the *x*-axis and the **dependent variable** up the *y*-axis. The independent variable is the quantity that, in an experiment or measurement, we start off by knowing, and the dependent variable is the quantity that we set out to measure or calculate. For example, if we ask 'how does the speed of this object change with time?' then time is the independent variable and speed the dependent variable. Similarly, if we ask 'how does the lifetime of a star depend on its mass?' then mass is the independent variable and we would probably plot a graph of mass along the *x*-axis and lifetime up the *y*-axis. But this is a convention not a hard-and-fast rule, and sometimes it is desirable to plot the axes the other way round. For example, Figure 6.13 shows how the temperature of the Earth's atmosphere depends on altitude. Altitude is the independent variable, but here it is plotted vertically because that is the direction in which altitude is measured.

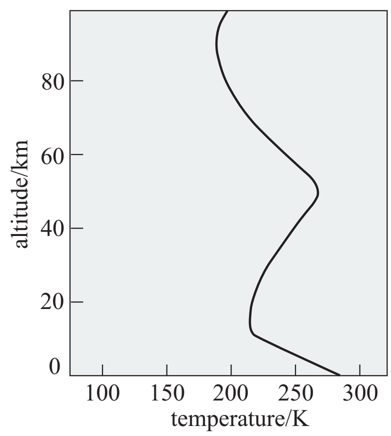


Figure 6.13 The variation of atmospheric temperature with height above the Earth's surface.

A graph can convey information in a much more compact way than a table of data. From Figure 6.11 you can deduce the object's speed at times that are not listed in Table 6.1. For example, by **interpolating** between the plotted points (reading

between them) you can see that at a time of 1.5 s the speed was 6 m s^{-1}, and similarly you can deduce that the object reached a speed of 8.5 m s^{-1} at a time t = 2.7 s. And by **extrapolating** (extending) the graph beyond the plotted points you can work out that at 4.5 s the speed would be 12 m s^{-1} (provided the object continued to move with the same acceleration).

The scales of the axes in Figure 6.11 were chosen to make plotting and reading the graph as easy as possible. This means using as much of the available space as possible while at the same time making sure that the graph-paper squares correspond to 'easy' numbers on the axes. A scale of one, two or five large graph-paper squares to each division on the axes will enable you to plot and read the graph easily. *Avoid* the temptation to have four, three or some other number of graph-paper squares per division, as the subdivisions on your scale will then fall in awkward places and you will need to do some arithmetic every time you plot or read a point.

6.6.2 Gradient of a graph

A graph's **gradient** is defined as shown in Figure 6.14 as

$$\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1} \tag{6.55a}$$

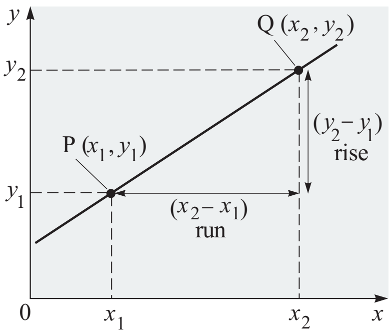


Figure 6.14 The gradient of a line joining two points P and Q.

A useful way to think of this is

$$\text{gradient} = \frac{\text{rise}}{\text{run}} \tag{6.55b}$$

In Figure 6.14, notice that the two points P and Q can lie anywhere on the line. To increase the precision of a calculation using values of x and y read from the graph, the two points should be as widely separated as possible. To work out the gradient, use values of x and y read from the scales of the graph (which will *not* in general be the same as distances measured off the graph with a ruler).

EXAMPLE 6.21

The graph in Figure 6.15 shows the distances to various earthquake monitoring stations plotted against the time at which waves arrived following a particular earthquake. Calculate the gradient of the graph and suggest a physical interpretation for its value.

Using the two points at time = 2 s, distance = 0 km and, time = 30 s, distance = 160 km, from Equation 6.55 we have

$$\text{gradient} = \frac{160\,\text{km}}{(30-2)\,\text{s}} = 5.7\,\text{km}\,\text{s}^{-1}$$

The gradient was found by dividing a distance by a time interval. In this graph, the gradient is equal to the speed of the waves.

The calculation of a gradient includes units. Often the gradient of a graph has a physical meaning, and in Example 6.22 the gradient of the graph was equal to the wave speed (5.7 km s^{-1}). The gradient of *any* graph of distance versus time will be interpreted as a speed, and in a graph of speed versus time the gradient is equal to the acceleration.

Notice that in Figure 6.15 the dependent variable is distance, but the graph is plotted as distance against time because that gives a more direct physical interpretation for the gradient.

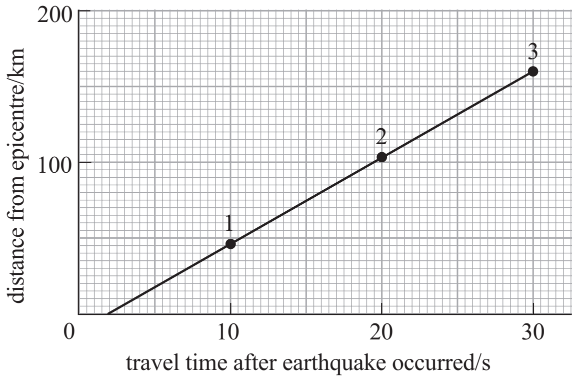


Figure 6.15 Graph of distance against time for waves produced in an earthquake.

QUESTION 6.34

What is the gradient of the graph shown in Figure 6.11? Suggest how the gradient of this graph can be interpreted physically.

Not all graphs are straight lines. For example, Figure 6.16 shows how the radius R of a hypothetical star might increase with time. It is still possible to calculate the gradient of a curved graph, as Figure 6.16 shows. Draw a straight line that just touches the curve at the point you are interested in, then calculate the gradient of that line. (In practice it is often difficult to judge exactly where to draw the line so the gradient cannot be calculated very precisely.)

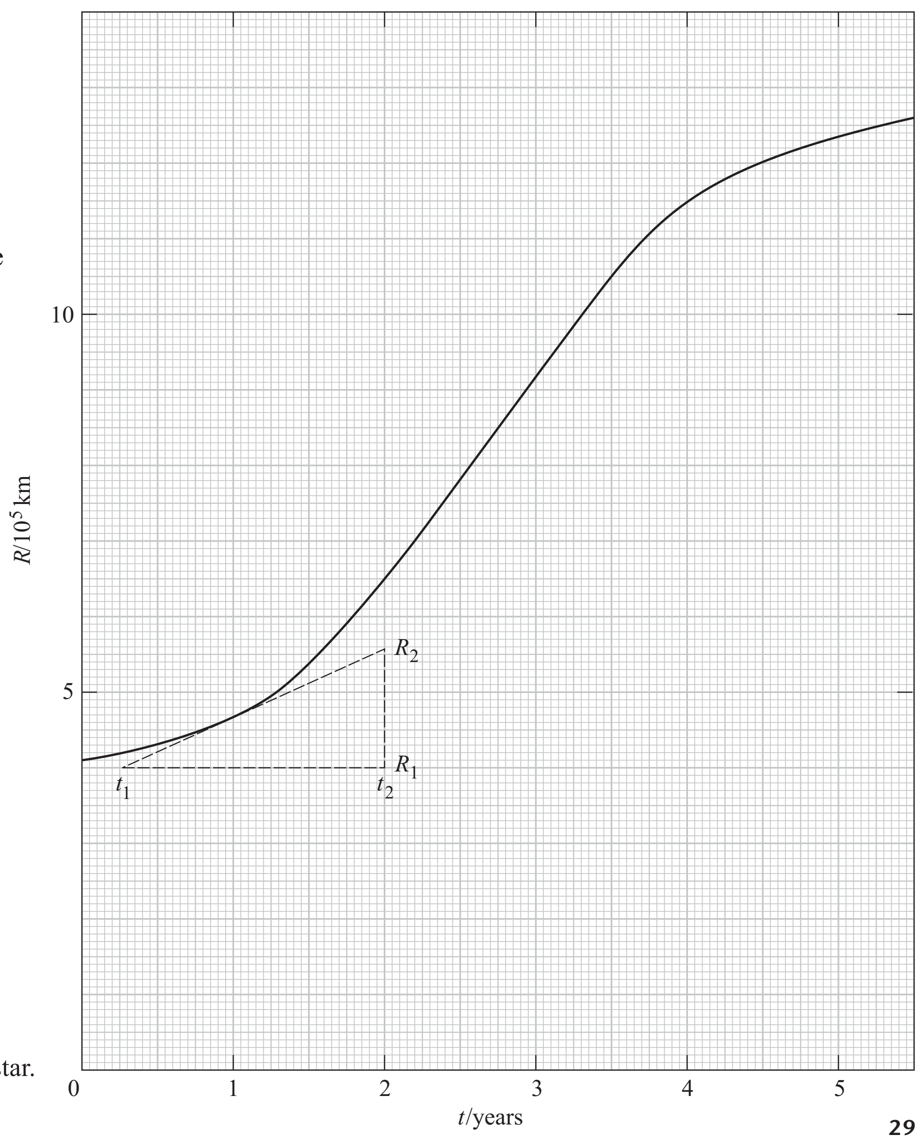


Figure 6.16 Expansion of a hypothetical star.

EXAMPLE 6.22

Use the gradient of the graph in Figure 6.16 to find the outward speed of the star's surface at a time of 1 year.

From Figure 6.16 and Equation 6.55, rise = $R_2 - R_1 = (5.6 - 4.0) \times 10^5$ km, and run = $t_2 - t_1 = (2.00 - 0.25)$ yr = 1.6×10^5 km/1.75 yr = 0.9×10^5 km yr^{-1}.

QUESTION 6.35

Calculate the gradient at $t = 4$ yr in Figure 6.16.

6.6.3 Equation of a straight line

If any quantity y is *directly proportional* to some other quantity x,

$$y \propto x, \text{ or } y = kx \tag{6.56}$$

then a graph of y against x is a straight line through the origin as shown in Figure 6.17. The *gradient* of such a line is always equal to the *constant of proportionality*, k. Starting from the *origin*, the 'rise' to any point is equal to its value of y, and the corresponding 'run' is equal to its value of x, and so from Equations 6.55 and 6.56,

$$\text{gradient} = y/x = k$$

In the example shown in Figure 6.17a, the gradient is 1.5 (e.g. rise/run = 6/4 = 1.5). Any value of y can be calculated from x using the equation

$$y = 1.5x$$

In Figure 6.17b, the line from Figure 6.17a has been shifted upwards by a distance labelled c (here $c = 2$). Now to calculate any value of y we need to use the

$$y = 1.5x + 2$$

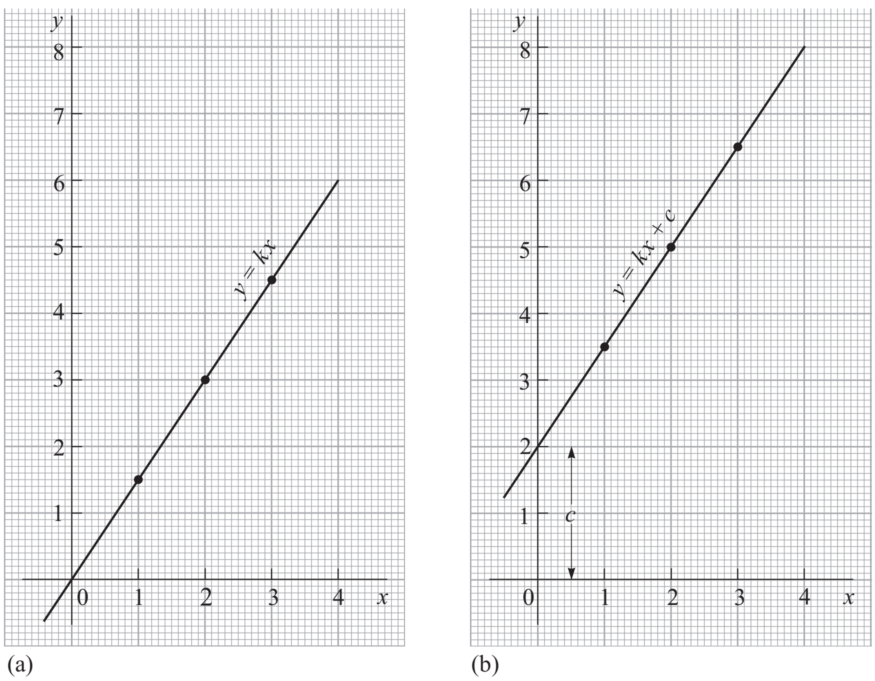


Figure 6.17 A graph of y against x when (a) $y = kx$ (b) $y = kx + c$.

The graph in Figure 6.17b still has a gradient of 1.5 since rise/run = (8 – 2)/4 = 6/4 = 1.5, but the line now cuts the y-axis above the origin at $y = 2$. Since this line does *not* pass through the origin, y is *not* proportional to x.

The **general equation of a straight line** is

$$y = kx + c \tag{6.57}$$

This tells us how to calculate the value of y corresponding to any value of x. And when those values are plotted on a graph, the gradient is equal to k, and the line cuts the y-axis above the origin when $y = c$; this value of y, corresponding to $x = 0$, is often known as the **intercept** on the y-axis. In many situations the intercept and gradient both have some physical meaning. For example, if an object's speed v at some time t is given by the equation

$$v = u + at = at + u$$

Comparison with Equation 6.57 shoes that a graph of v plotted against t has gradient equal to the *acceleration*, a, and intercepts the y-axis at u, which is the speed when $t = 0$.

6.6.4 Logarithmic scales

Sometimes we need to plot graphs covering a huge range of values. For example, the *temperature* in the outer regions of the Sun varies with height above the surface as shown (approximately) in Table 6.2. If we want to show details of variation close to the surface we might plot a graph with axes as shown in Figure 6.18a, but then points representing the larger values would need a sheet of paper at least one hundred times wider and one thousand times longer. If we wanted to include all the values on a graph of reasonable size, then we might plot a graph with axes as shown in Figure 6.18b. Now we can plot the larger values, but the smaller ones are crammed so close to the origin that they cannot be seen. In either case, a *linear scale* is unsatisfactory.

Table 6.2 Variation of temperature with height in the outer regions of the Sun.

Height above surface, h/km	Temperature, T/K
1.0×10^2	6.0×10^3
3.0×10^2	5.0×10^3
1.0×10^3	6.0×10^3
3.0×10^3	2.0×10^5
1.0×10^4	1.0×10^6
1.0×10^5	2.0×10^6

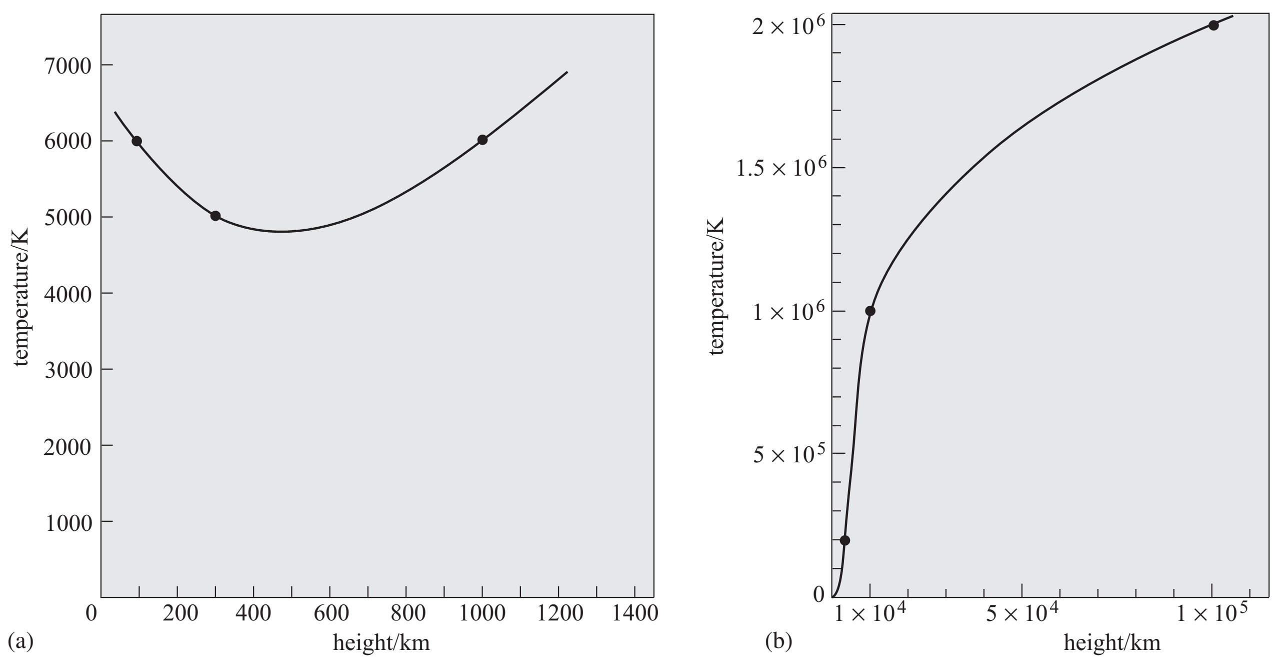


Figure 6.18 Two possible choices of axes for plotting the values from Table 6.2.

Table 6.3 Logarithms of the values listed in Table 6.2.

$\log_{10}(h/\text{km})$	$\log_{10}(T/\text{K})$
2.0	3.8
2.5	3.7
3.0	3.8
3.5	5.3
4.0	6.0
5.0	6.3

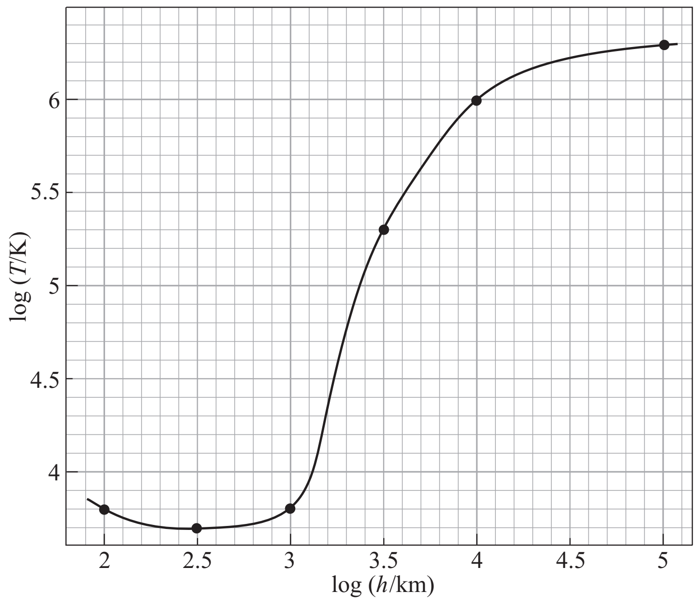


Figure 6.19 A logarithmic graph showing the variation of temperature with height in the outer regions of the Sun.

To display values where the largest is more than few tens of times bigger than the smallest, we often plot a graph with a **logarithmic scale** (often abbreviated to 'log scale'). There are two ways to construct a logarithmic scale, which both boil down to exactly the same thing.

One way to make a logarithmic scale is to label the axes of a linear scale using the *common logarithms* of the quantities to be plotted. Table 6.3 lists the logarithms of the values given in Table 6.2. The logarithms cover a much smaller range than the values themselves. Figure 6.19 shows a logarithmic graph of the data from Table 6.3.

The other way to make a log scale is to label the axes using *powers of ten*, in such a way that the powers increase in equal steps. When plotting a graph in this way, we can use logarithmic graph paper as shown in Figure 6.20. The beginning of each 'cycle' is

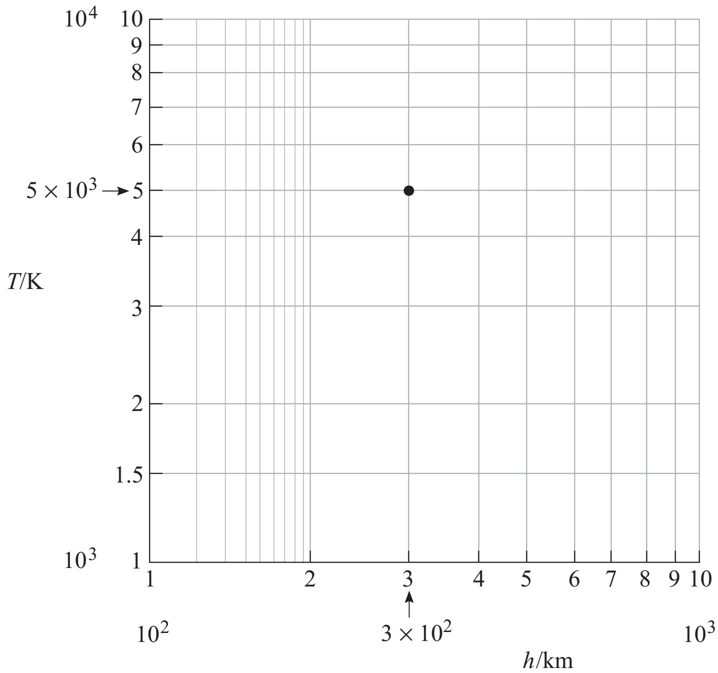


Figure 6.20 Plotting a point on logarithmic graph paper.

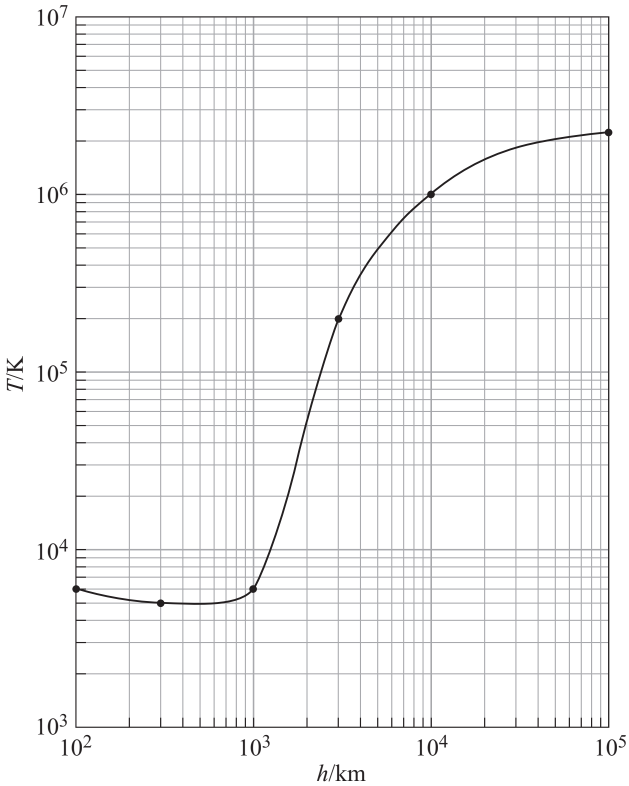


Figure 6.21 A graph of data from Table 6.2 plotted on logarithmic graph paper.

labelled with a whole-number power of ten, and the grid shows where to plot intermediate values. Figure 6.20 shows how to plot the point for $h = 3.0 \times 10^2$ km, $T = 5.0 \times 10^3$ K on logarithmic graph paper, and Figure 6.21 shows a graph of the data from Table 6.2 plotted in this way.

Notice that Figure 6.21 shows a graph with exactly the same shape as that in Figure 6.19. Either way of plotting the graph is equally good, because both display the information in the same way.

Logarithmic graphs can involve negative *powers of ten* (and hence negative *logarithms*) as well as positive ones. There is an example in Figure 6.22, which shows the *frequencies* and *wavelengths* of *electromagnetic radiation*. The numbers on such a scale get smaller and smaller with each power of ten, but never quite reach zero, so the axes of a logarithmic graph do not include the *origin*.

The examples in Figures 6.19 and 6.21 are **log–log graphs**; both axes have logarithmic scales. Sometimes it can be useful to plot a log–linear graph, in which only one axis has a logarithmic scale and the other has an ordinary *linear scale*. Such a **log–linear graph** might be appropriate if only one set of values covers a large range.

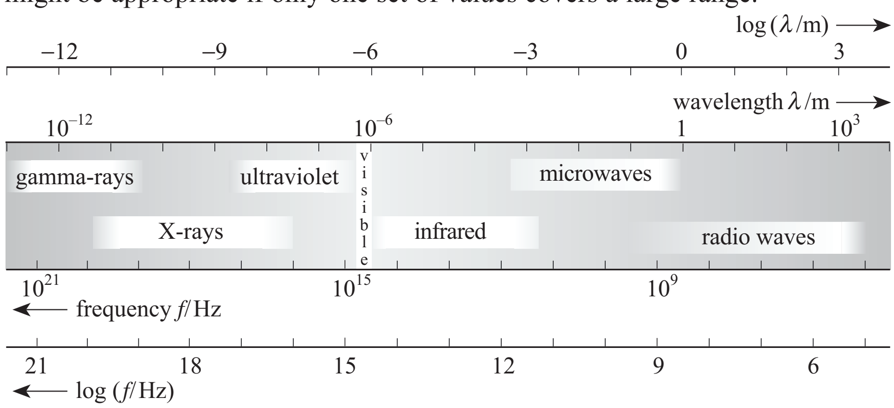


Figure 6.22 The electromagnetic spectrum.

QUESTION 6.36

Table 6.4 lists some information about the planets in the Solar System: their approximate distances from the Sun and their orbital periods (the time to travel once around an orbit). Plot two logarithmic graphs of these values, using the grids in Figure 6.23. (*Hint*: start by making a table of $\log_{10}(d/10^6\,\text{km})$ and $\log_{10}(T/\text{yr})$.)

Table 6.4 The distances and orbital periods of planets in the Solar System.

Planet	Distance from Sun $d/10^6$ km	Orbital period T/yr
Mercury	58	0.24
Venus	1.1×10^2	0.62
Earth	1.5×10^2	1.0
Mars	2.3×10^2	1.9
Jupiter	7.8×10^2	12
Saturn	1.4×10^3	29
Uranus	2.9×10^3	84
Neptune	4.5×10^3	1.6×10^2
Pluto	5.9×10^3	2.5×10^2

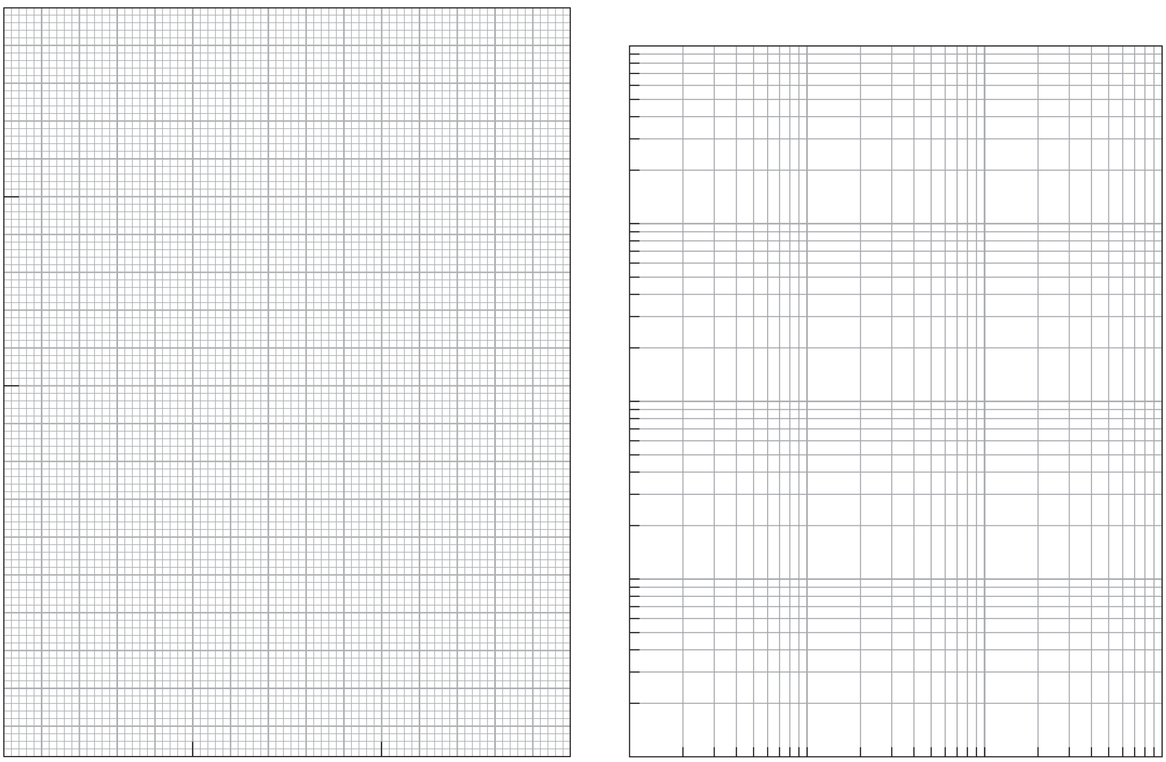


Figure 6.23 Graph paper for use with Question 6.36.

6.6.5 Reading log scales

To read information from a graph plotted on logarithmic graph paper, simply use the grid lines and the 'powers of ten' labels. If the graph is plotted using logarithms of values plotted on a linear scale, then read the graph in the normal way then find the *antilog* of the values. For example, if you wanted to use Figure 6.19 to find the height at which the temperature was 1×10^5 K, you would first find

$$\log_{10}(T/\text{K}) = \log(1 \times 10^5) = 5.0$$

then read the graph as shown in Figure 6.24. Here $\log_{10}(h/\text{km}) \approx 3.44$

so $h/\text{km} = \text{antilog}_{10}(3.44) = 2.75 \times 10^3$

i.e. $h = 2.75 \times 10^3$ km

You will probably come across logarithmic graphs plotted using powers of ten, but where divisions are not shown. Figure 6.24 shows such a graph based on Figure 6.21. This makes it look quite tricky to read values lying between the powers of ten.

One way to read a graph such as Figure 6.24 is to imagine the axes labelled with logarithms as in Figure 6.19. Then estimate roughly where your chosen point lies in relation to whole-number powers. For example, if you needed to use Figure 6.24 to find the height at which the temperature was 1×10^5 K, you might judge that the relevant point lay about half of the way along from 10^3 km to 10^4 km. So you would then say that

$$\log_{10}(h/\text{km}) \approx 3.5$$

and, using antilogs as above, could work out that

$$h/\text{km} \approx \text{antilog}(3.5) \approx 3 \times 10^3$$

$$h \approx 3 \times 10^5 \text{ km}$$

This is of course a less precise answer than you would get if the grid lines were shown.

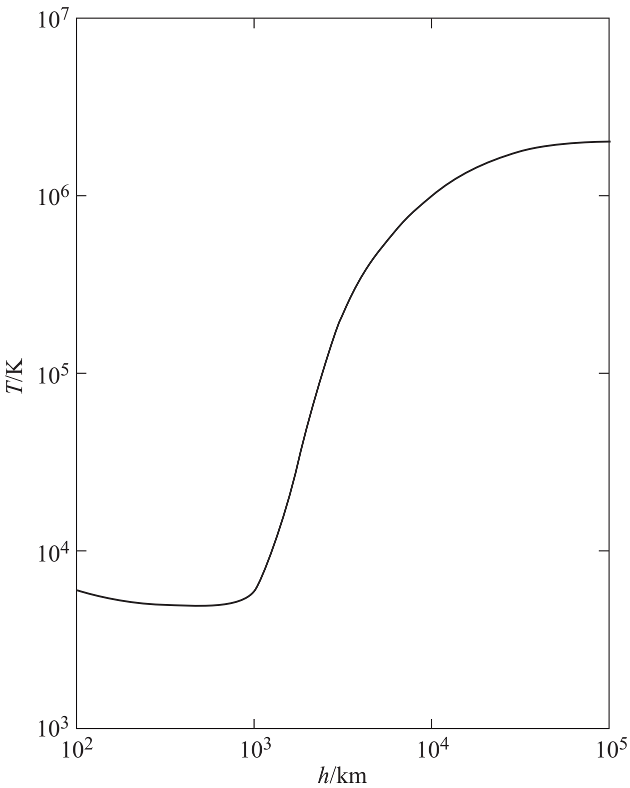


Figure 6.24 A logarithmic graph drawn without grid lines, or sub-divisions on the x-axis.

Another way to read graphs such as that in Figure 6.24 is to picture the grid lines from Figure 6.21. Notice that the grid lines on logarithmic graph paper get closer together as you move towards the higher power of ten, and the '3' falls about half way along. This gives us a quick approximate way to read the scale. A point lying mid-way between two powers of ten can be read as '$3 \times \ldots$' the lower power, and other points can likewise be estimated. '$1.5 \times \ldots$' lies about one-fifth of the way along, '$2 \times \ldots$' lies about one-third of the way along, and '$5 \times \ldots$' lies about three-quarters of the way towards the higher power.

QUESTION 6.37

On Figure 6.19, find the point on the x-axis corresponding to a height of 5×10^3 km. Then find the temperature at that height.

QUESTION 6.38

On Figure 6.24, mark the point corresponding to about 3×10^4 km. Then read the approximate temperature at that height.

6.7 Answers and comments for Topic 6

QUESTION 6.1

(i) $15a^7$ (ii) $3b^3$ (iii) 6 (iv) $16d^8$ (i.e. $2^4 \times d^8$)

QUESTION 6.2

(i) $2a^{-4}$ (ii) $b^{-2}/3$

QUESTION 6.3

(i) 0.008 (ii) 0.028 57…

QUESTION 6.4

(i) $6a^{3/4}$ (i.e. $2a^{3/2} \times 3a^{1/2}$) (ii) $3\sqrt{b}$ (iii) $2c/d$

QUESTION 6.5

(i) 3.162… (ii) 4913 (iii) 4.429…

QUESTION 6.6

(i) 1.5×10^{11} m (ii) 1.60×10^{-19} C

QUESTION 6.7

(i) 3.03×10^{-19} (ii) 6.17×10^{14}

QUESTION 6.8

(i) 4 (ii) −5 (iii) 0

QUESTION 6.9

(i) 0.3010… (ii) 0.501 371… (iii) −0.204…

QUESTION 6.10

The answers to this question are exactly the same as those for Question 6.9.

(i) If $10^x = 2$ then $x = \log_{10}(2) = 0.3010…$ (ii) 0.501 371… (iii) −0.204…

QUESTION 6.11

(i) 3.162… (ii) 16.982… (iii) 0.0588…

QUESTION 6.12

(i) two (ii) two (iii) two (iv) three (v) four

QUESTION 6.13

On a calculator,

Speed = $241 \times 10^6\,\mathrm{m}/(33 \times 3600\,\mathrm{s}) = 2.028\ 62 \times 10^3\,\mathrm{m\,s^{-1}}$

The time, 33 hours, has only two significant figures so the final answer only has two: the average speed is $2.0 \times 10^3\,\mathrm{m\,s^{-1}}$.

QUESTION 6.14

(i) $10^{-23}\,\mathrm{J\,K^{-1}}$ (ii) 10 (i.e. 10^1) (iii) $10^{30}\,\mathrm{kg}$ (iv) $10^{12}\,\mathrm{m}$

QUESTION 6.15

Distance $\approx (3 \times 3 \times 10^8 \times 10^7)\,\mathrm{m} = 9 \times 10^{15}\,\mathrm{m}$.

QUESTION 6.16

Write $ma = F$ then divide both sides by m to get $a = F/m$.

QUESTION 6.17

First use one of the two methods of Example 6.8 to get r^2 on its own.

Either start by taking the reciprocal of both sides to get $r^2/GMm = 1/F$, then multiply by G, M and m to get $r^2 = GMm/F$,

or multiply both sides by r^2 to get $r^2F = GMm$ then divide by F to get $r^2 = GMm/F$.

Finally take the square root of both sides to get

$$r = \sqrt{\frac{GMm}{F}}$$

QUESTION 6.18

Distance $x = 2\pi r$ so $v = 2\pi r/T$.

QUESTION 6.19

The mass m is the unwanted quantity. First make m the subject of the density equation: $m = \rho V$. Then substitute for m in the equation for internal energy: $\Delta q = \rho V c\,\Delta T$.

QUESTION 6.20

Pressure $p \propto n$ and $p \propto T$. (Notice that $p \propto k$ is *not* a correct answer. It is a meaningless statement because k is a constant.)

QUESTION 6.21

(i) $y \propto 1/x$ (ii) $y \propto x^2$ (iii) $y \propto 1/x^2$

QUESTION 6.22

We can first write $L \propto kAT^4$ or $L = kAT^4$ where k is some constant. Then, because $A = Kr^2$ where A is some constant, we can also write $L = Kkr^2T^4$ which becomes $L \propto r^2T^4$.

QUESTION 6.23

First write $t^2 = kr^3$. Then use t_1 and r_1 and t_2 and r_2 to represent quantities for the Earth and for Jupiter respectively.

$$t_1^2 = kr_1^3 \text{ and } t_2^2 = kr_2^3$$

Either make k the subject to get

$$\frac{t_2^2}{r_2^3} = \frac{t_1^2}{r_1^3}$$

or divide one version by the other to get

$$\left(\frac{t_2}{t_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$$

In either case,

$$\begin{aligned} t_2 &= t_1\sqrt{\left(\frac{r_2}{r_1}\right)^3} \\ &= t_1\sqrt{\left(\frac{5.2r_1}{r_1}\right)^3} \\ &= 1\,\text{year} \times \sqrt{5.2^3} \\ &= 12\,\text{years} \end{aligned}$$

QUESTION 6.24

(i) Putting $D = 2r$ in Equation 6.38, $A_{sph} = 4\pi(D/2)^2 = 4\pi D^2/4 = \pi D^2$.

(ii) With $D = 2r$ in Equation 6.39, $V_{sph} = 4\pi(D/2)^3/3 = 4\pi D^3/(3 \times 8) = \pi D^3/6$.

QUESTION 6.25

From Equation 6.35, $C = 2\pi \times 1.50 \times 10^{11}\,\text{m} = 9.42 \times 10^{11}\,\text{m}$.

QUESTION 6.26

(i) Using Equation 6.38, $A = 4\pi \times (6.96 \times 10^8\,\text{m})^2 = 6.09 \times 10^{18}\,\text{m}^2$.

(ii) Using Equation 6.39, $V = 4\pi \times (6.96 \times 10^8\,\text{m})^3/3 = 1.41 \times 10^{27}\,\text{m}^3$.

QUESTION 6.27

60° is one-sixth of a complete turn, so from Equation 6.41, $60° = \pi/3$ radians. From Equation 6.43, we have $60 \times (2\pi/360)$ radians = 1.047 radians.

QUESTION 6.28

From Equation 6.41, we know $\pi/4$ radians must correspond to one-eighth of a complete turn, that is 45°. Alternatively, from Equation 6.42, we have $(360°/2\pi) \times \pi/4 = 45°$.

QUESTION 6.29

1 arcsec = 1 arcmin/60 = 1°/3600. So 1 arcsec = (2π/360) × (1/3600) radians = 4.847×10^{-6} radians.

QUESTION 6.30

Suppose the unknown side is y. Then from Equation 6.44, $y^2 = h^2 - x^2$ $= (7.2\,\text{cm})^2 - (6.5\,\text{cm})^2 = 51.84\,\text{cm}^2 - 42.25\,\text{cm}^2 = 9.59\,\text{cm}^2$ so

$$y = \sqrt{9.59\,\text{cm}^2} = 3.1\,\text{cm}$$

QUESTION 6.31

The distance is the hypotenuse, h, of the right-angled triangle in Figure 6.7. We know the two other sides so we can use Equation 6.46:

$$x/h = \cos\theta$$

so $\quad x = h\cos\theta$

$$h = x/\cos\theta = 20\,\text{m}/\cos 60° = 40\,\text{m}$$

Or use Equation 6.45 and the answer from Example 6.17:

$$y/h = \sin\theta$$

$$h = y/\sin\theta = 35\,\text{m}/\sin 60° = 40\,\text{m}$$

QUESTION 6.32

Use the same method as in Question 6.31, but switch your calculator into radian mode. Either use Equation 6.46:

$$x/h = \cos\theta$$

so $\quad x = h\cos\theta$

$$h = x/\cos\theta = 20\,\text{m}/\cos(\pi/3\,\text{rad}) = 40\,\text{m}$$

Or use Equation 6.45 and the answer from Example 6.17:

$$y/h = \sin\theta$$

$$h = y/\sin\theta = 35\,\text{m}/\sin(\pi/3\,\text{rad}) = 40\,\text{m}.$$

QUESTION 6.33

From Equation 6.49 and Example 6.20, $\theta = 8.7 \times 10^{-3}$ rad and distance $r = d/\alpha = 3.5 \times 10^6\,\text{m}/8.7 \times 10^{-3}\,\text{rad} = 4 \times 10^8\,\text{m}$.

QUESTION 6.34

If we choose the points $t_1 = 0.0$ s, $\upsilon_1 = 3.0\,\text{m s}^{-1}$, and $t_2 = 5.0$ s, $\upsilon_2 = 13.0\,\text{m s}^{-1}$, we have from Equation 6.55:

$$\text{gradient} = (13.0 - 3.0)\,\text{m s}^{-1}/(5.0 - 0.0)\,\text{s} = 10.0\,\text{m s}^{-1}/5.0\,\text{s} = 2.0\,\text{m s}^{-2}$$

The gradient here is equal to the object's *acceleration*: it is found by dividing a change in speed by the time interval over which it occurred.

QUESTION 6.35

From Figure 6.25, rise = $(13.6 - 9.0) \times 10^5$ km = 4.6×10^5 km,
run = $(5.50 - 2.15)$ yr = 3.35 yr.

From Equation 6.55, gradient = 4.6×10^5 km/3.35 yr ≈ 1.4×10^5 km yr^{-1}. (Any value close to 1.4×10^5 km yr^{-1} is fine, because your exact value will depend on where you drew the line.)

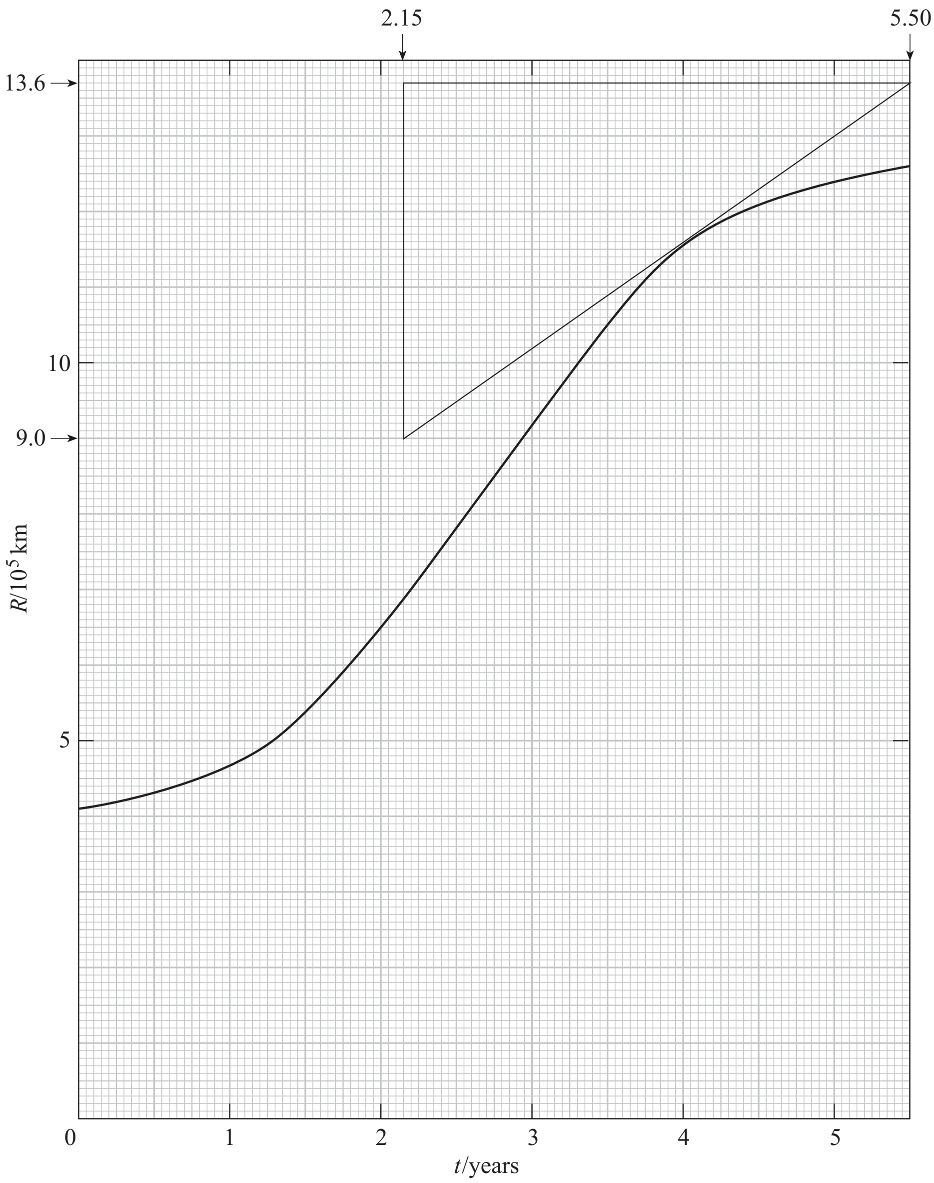


Figure 6.25 Graph for the answer to Question 6.35.

QUESTION 6.36

See Figure 6.26. Table 6.5 lists the logarithms for plotting the graph in Figure 6.26.

Table 6.5 Logarithms of the values listed in Table 6.4.

Planet	$\log_{10}(d/10^6\,\text{km})$	$\log_{10}(T/\text{yr})$
Mercury	1.76	–0.620
Venus	2.04	–0.208
Earth	2.18	0.00
Mars	2.36	0.279
Jupiter	2.89	1.08
Saturn	3.15	1.46
Uranus	3.46	1.92
Neptune	3.65	2.20
Pluto	3.77	2.40

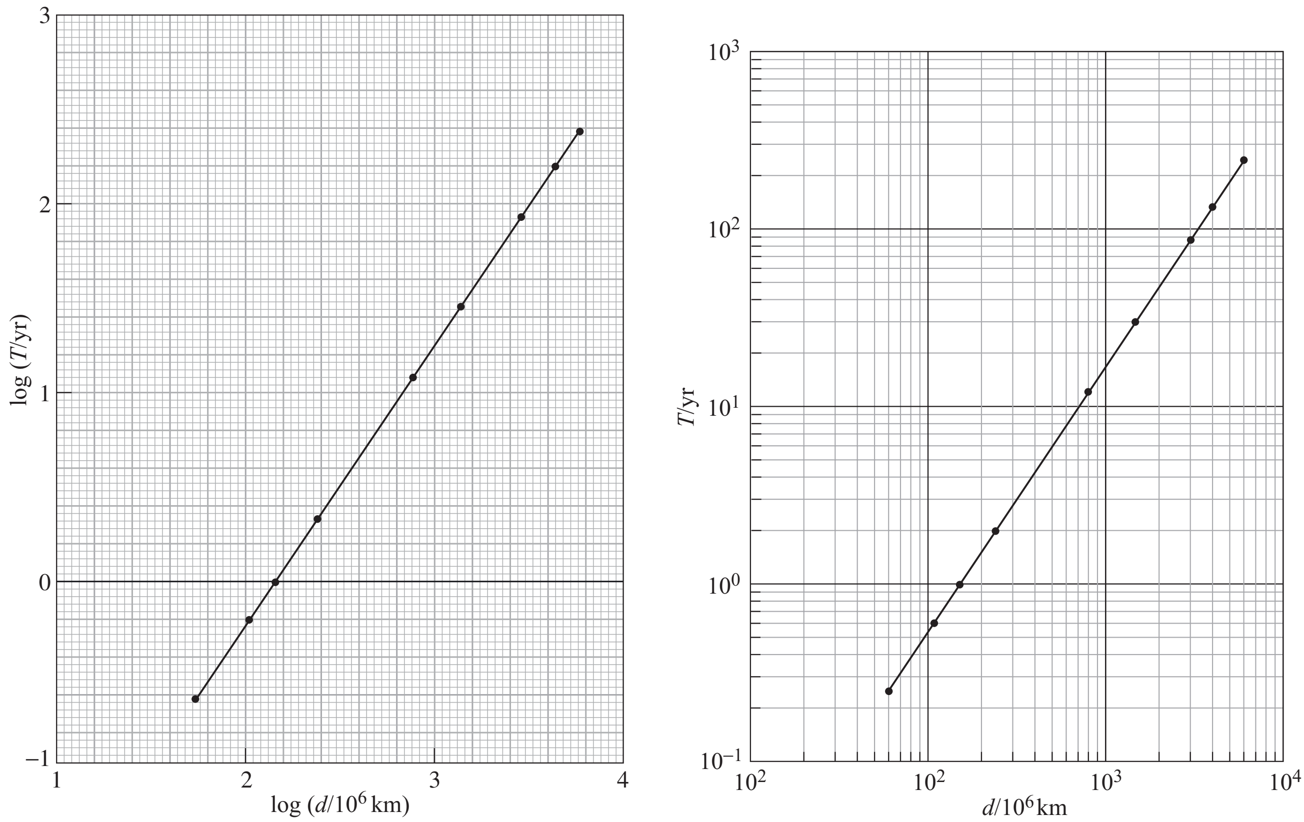


Figure 6.26 The answer to Question 6.36: (left) logs of the values plotted on ordinary graph paper (right) values plotted on logarithmic graph paper

QUESTION 6.37

$\log_{10}(h/\text{km}) = \log_{10}(5 \times 10^3) = 3.7$. From the graph, the temperature at this height has $\log(T/\text{K}) = 5.6$, so $T/\text{K} = \text{antilog}_{10}(5.6) \approx 4.0 \times 10^5$, therefore $T \approx 4.0 \times 10^5$ K.

QUESTION 6.38

The point for 3×10^4 km lies about half-way along from 1×10^4 km. From Figure 6.27, the point for the corresponding temperature is about one-fifth of the way up from 1×10^6 K. so the temperature is about 1.5×10^6 K.

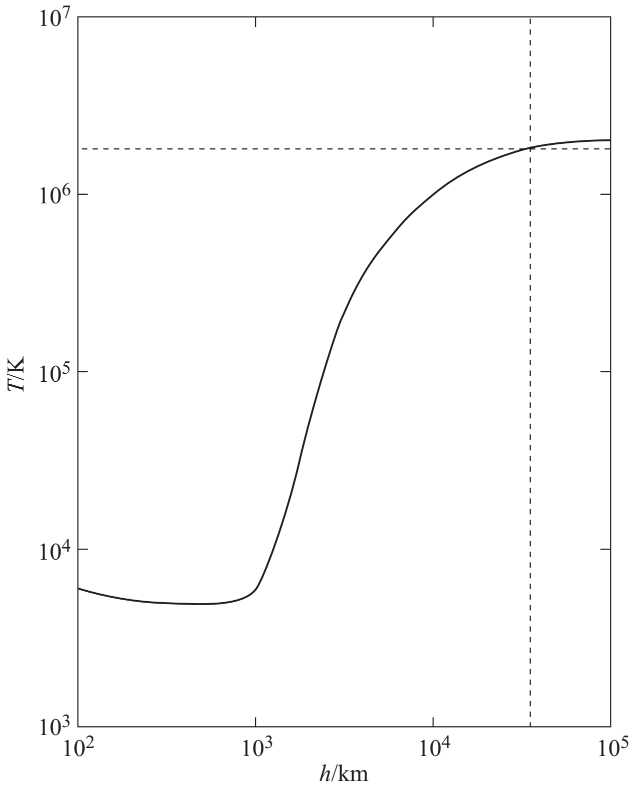


Figure 6.27 Reading the graph to answer Question 6.38.

TOPIC 7 USEFUL TABLES AND DATA

7.1 The Periodic Table

	Group I	Group II												Group III	IV	V	VI	VII	VIII
Period 1			1 H																2 He
Period 2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
Period 3	11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
Period 4	19 K	20 Ca		21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
Period 5	37 Rb	38 Sr		39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
Period 6	55 Cs	56 Ba		71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
Period 7	87 Fr	88 Ra		103 Lr	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110	111	112	113	114	115	116	117	118

transition elements

typical elements

lanthanides	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb
actinides	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No

Table 7.1 The Periodic Table.

7.2 The Greek alphabet

Table 7.2 The Greek alphabet.

Name	Lower case	Upper case	Name	Lower case	Upper case
Alpha	α	A	Nu (new)	ν	N
Beta (bee-ta)	β	B	Xi (cs-eye)	ξ	Ξ
Gamma	γ	Γ	Omicron	o	O
Delta	δ	Δ	Pi (pie)	π	
Epsilon	ε	E	Rho (roe)	ρ	P
Zeta (zee-ta)	ζ	Z	Sigma	σ	Σ
Eta (ee-ta)	η	H	Tau	τ	T
Theta (thee-ta – 'th' as in theatre)	θ	Θ	Upsilon	υ	Y
Iota (eye-owe-ta)	ι	I	Phi (fie)	ϕ	Φ
Kappa	κ	K	Chi (kie)	χ	X
Lambda (lam-da)	λ	Λ	Psi (ps-eye)	ψ	Ψ
Mu (mew)	μ	M	Omega (owe-me-ga)	ω	Ω

7.3 Selected physical constants and unit conversions

Table 7.3 SI fundamental and derived units.

Quantity	Unit	Abbreviation	Equivalent units
mass	kilogram	kg	
length	metre	m	
time	second	s	
temperature	kelvin	K	
angle	radian	rad	
area	square metre	m^2	
volume	cubic metre	m^3	
speed, velocity	metre per second	$m\,s^{-1}$	
acceleration	metre per second squared	$m\,s^{-2}$	
density	kilogram per cubic metre	$kg\,m^{-3}$	
frequency	hertz	Hz	(cycles) s^{-1}
force	newton	N	$kg\,m\,s^{-2}$
pressure	pascal	Pa	$kg\,m^{-1}\,s^{-2}$, $N\,m^{-2}$
energy	joule	J	$kg\,m^2\,s^{-2}$
power	watt	W	$kg\,m^2\,s^{-3}$, $J\,s^{-1}$
specific heat capacity	joule per kilogram kelvin	$J\,kg^{-1}\,K^{-1}$	$m^2\,s^{-2}\,K^{-1}$
thermal conductivity	watt per metre kelvin	$W\,m^{-1}\,K^{-1}$	$m\,kg\,s^{-3}\,K^{-1}$

Table 7.4 Selected physical constants and preferred values.

Quantity	Symbol	Value
speed of light in a vacuum	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Planck constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Boltzmann constant	k	$1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{N\,m^{2}\,kg^{-2}}$
Stefan–Boltzmann constant	σ	$5.67 \times 10^{-8}\,\mathrm{W\,m^{2}\,K^{-4}}$
Avogadro constant	N_{A}	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
molar gas constant	R	$8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$
charge of electron	e	1.60×10^{-19} C (negative charge)
mass of proton	m_{p}	$1.67 \times 10^{-27}\,\mathrm{kg}$
mass of electron	m_{e}	$9.11 \times 10^{-31}\,\mathrm{kg}$
Astronomical quantities:		
mass of the Sun	$M_{\odot}$	$1.99 \times 10^{30}\,\mathrm{kg}$
radius of the Sun	$R_{\odot}$	$6.96 \times 10^{8}\,\mathrm{m}$
photospheric temperature of the Sun	$T_{\odot}$	5770K
luminosity of the Sun	$L_{\odot}$	$3.84 \times 10^{26}\,\mathrm{W}$
astronomical unit	AU	$1.50 \times 10^{11}\,\mathrm{m}$

Table 7.5 Some useful conversions from alternative unit systems to SI units.

Quantity	Unit	SI equivalent
angle	1 degree	(π/180) radians
pressure	1 bar	10^{5} pascals
temperature	1 °C	1 K
energy	1 erg	10^{-7} joules
	1 electron volt	$1.60 \times 10^{-19}\,\mathrm{J}$
	1 ton of TNT	$4.18 \times 10^{9}\,\mathrm{J}$
length	1 foot	0.305 m
	1 mile	1.61 km
area	1 square inch	$6.45 \times 10^{-4}\,\mathrm{m^{2}}$
	1 square mile	$2.59 \times 10^{6}\,\mathrm{m^{2}}$
mass	1 pound	0.454 kg
speed, velocity	1 mile per hour	$0.447\,\mathrm{m\,s^{-1}}$